

Blob homology, part II

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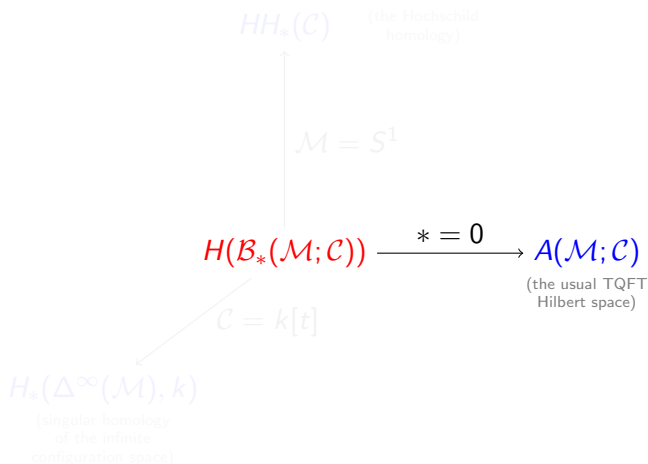
Homotopy Theory and Higher Algebraic Structures, UC
Riverside, November 10 2009
<http://tqft.net/UCR-blobs1>

Outline

1 Overview

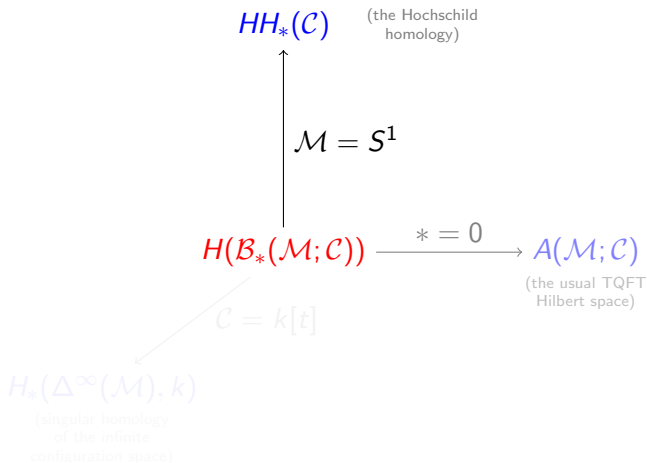
What is *blob homology*?

The blob complex takes an n -manifold \mathcal{M} and an ' n -category with strong duality' \mathcal{C} and produces a chain complex, $\mathcal{B}_*(\mathcal{M}; \mathcal{C})$.



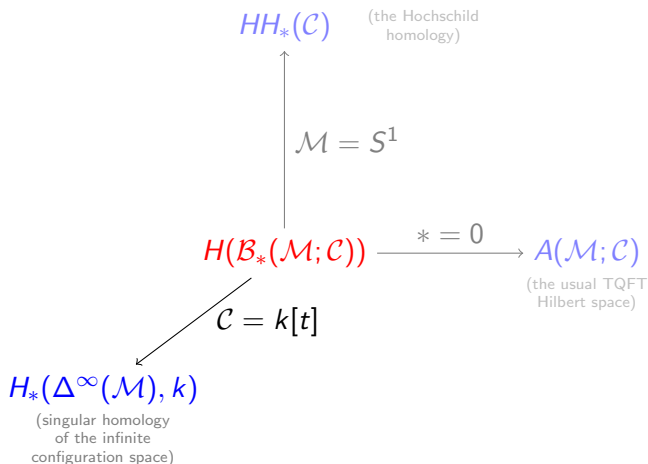
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n -categories

Defining n -categories is fraught with difficulties

I'm not going to go into details; I'll draw 2-dimensional pictures, and rely on your intuition for pivotal 2-categories.

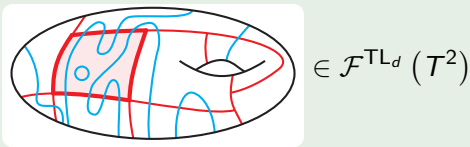
- Kevin's talk (part III) will explain the notions of 'topological n -categories' and ' A_∞ n -categories'.
- Defining n -categories: a choice of 'shape' for morphisms.
- We allow all shapes! A vector space for every ball.
- 'Strong duality' is integral in our definition.

Fields and pasting diagrams

Pasting diagrams

Fix an n -category with strong duality \mathcal{C} . A *field* on \mathcal{M} is a pasting diagram drawn on \mathcal{M} , with cells labelled by morphisms from \mathcal{C} .

Example ($\mathcal{C} = \text{TL}_d$ the Temperley-Lieb category)



Given a field on a ball, we can evaluate it to a morphism. We call the kernel the *null fields*.

$$\text{ev} \left(d \left(\text{Diagram 1} - \text{Diagram 2} \right) \right) = 0$$

The diagram shows the evaluation of a difference of two fields on a ball. The first field (Diagram 1) consists of a red horizontal line and two blue strands that cross it. The second field (Diagram 2) consists of a red horizontal line and two blue strands that do not cross it. The evaluation of their difference is zero.

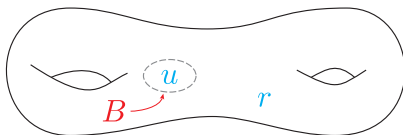
Definition of the blob complex, $k = 0, 1$

Motivation

A *local* construction, such that when \mathcal{M} is a ball, $\mathcal{B}_*(\mathcal{M}; \mathcal{C})$ is a resolution of $A(\mathcal{M}, ; \mathcal{C})$.

$$\mathcal{B}_0(\mathcal{M}; \mathcal{C}) = \mathcal{F}(\mathcal{M}), \text{ arbitrary fields on } \mathcal{M}.$$

$$\mathcal{B}_1(\mathcal{M}; \mathcal{C}) = \left\{ (B, u, r) \mid \begin{array}{l} B \text{ an embedded ball} \\ u \in \mathcal{F}(B) \text{ in the kernel} \\ r \in \mathcal{F}(\mathcal{M} \setminus B) \end{array} \right\}.$$

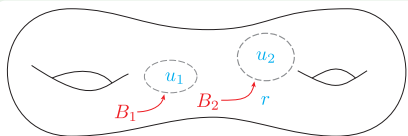


$$d_1 : (B, u, r) \mapsto u \circ r \quad \mathcal{B}_0 / \text{im}(d_1) \cong A(\mathcal{M}; \mathcal{C})$$

Definition, $k = 2$

$$\mathcal{B}_2 = \mathcal{B}_2^{\text{disjoint}} \oplus \mathcal{B}_2^{\text{nested}}$$

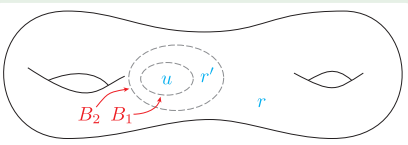
$$\mathcal{B}_2^{\text{disjoint}} =$$



$$u_i \in \ker \text{ev}_{B_i}$$

$$d_2 : (B_1, B_2, u_1, u_2, r) \mapsto (B_2, u_2, r \circ u_1) - (B_1, u_1, r \circ u_2)$$

$$\mathcal{B}_2^{\text{nested}} =$$



$$u \in \ker \text{ev}_{B_1}$$

$$d_2 : (B_1, B_2, u, r', r) \mapsto (B_2, u \circ r', r) - (B_1, u, r \circ r')$$