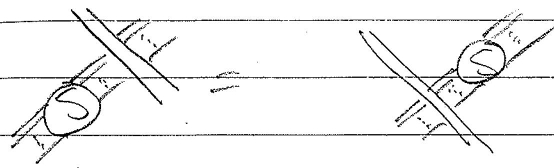
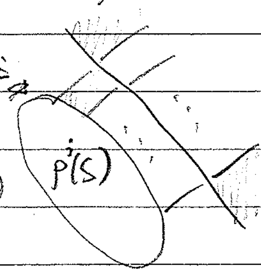
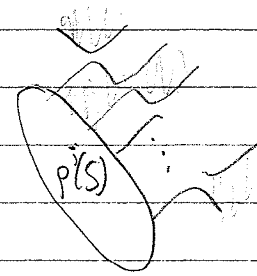
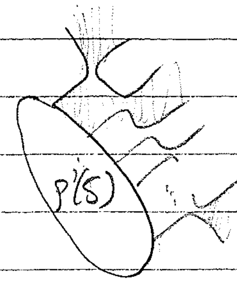
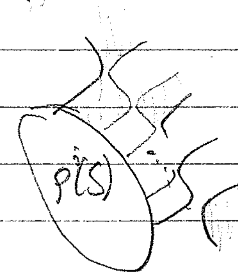
Let S generate $D(2n)$ as a planar algebra

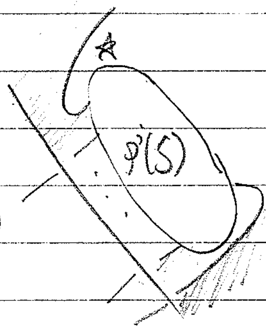
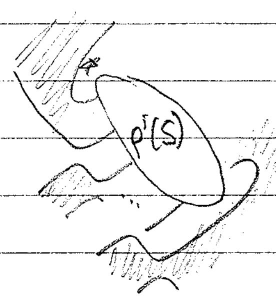
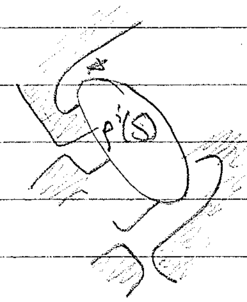
<u>Claim:</u>

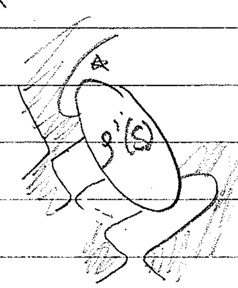


~~Pf~~:

(1)  $= q^{2n}$  $+ q^{2n-2}$ 

$+ \cdots + q^{-2n}$ 

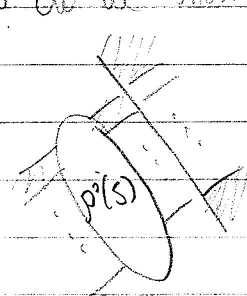
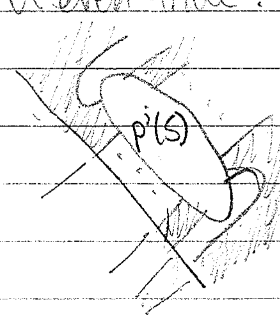
(2)  $= q^{2n}$  $+ q^{2n-2}$ 

$+ \cdots + q^{-2n}$ 

(3) $-(q^2+q^{-2}) = 2\cos\left(\pi/4n+2\right) \Rightarrow q^2 = -e^{i\pi/4n+2} \Rightarrow (q^2)^{2n+1} = (-e^{i\pi/4n+2})^{2n+1} =$
$= (-e^{i\pi/2}) = -i$

(4) $p(S) = -S$: how do we know this? is it even true?

<u>Lemma:</u>

 $= i$ 

Pf:

$$4n+2\left\{\overbrace{\begin{bmatrix} -q^2-q^{-2} & 1 & 0 & \cdots & 0 & -1 \\ 1 & -q^2-q^{-2} & 1 & 0 & \cdots & 0 \\ 0 & 1 & -q^2-q^{-2} & 1 & 0 & \cdots \\ \vdots & 0 & 1 & \ddots & \ddots & \ddots \\ \vdots & \vdots & 0 & & \ddots & \\ 0 & \vdots & \vdots & & & \\ -1 & 0 & \vdots & & & \end{bmatrix}}^{4n+2}\right. \begin{bmatrix} q^{2n} \\ q^{2n-2} \\ \vdots \\ q^{-2n} \\ i q^{2n} \\ i q^{2n-2} \\ \vdots \\ i q^{-2n} \end{bmatrix} =$$

(4)

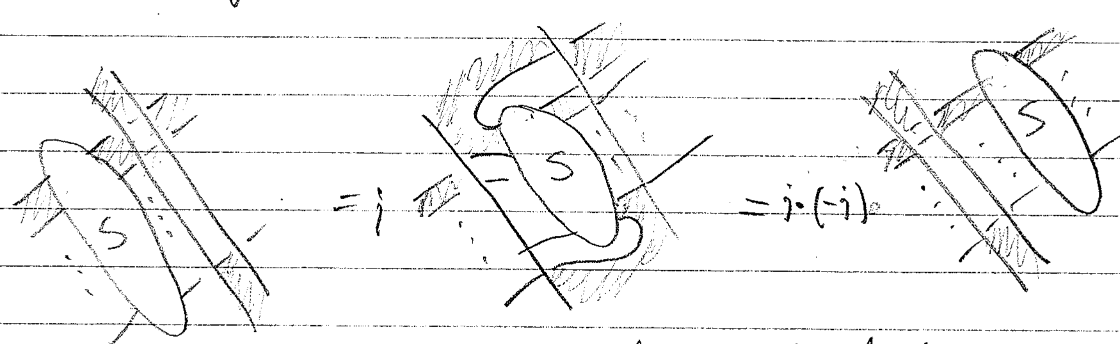
$$= \begin{bmatrix} -q^{2n+2} - q^{2n-2} + q^{2n-2} - i q^{-2n} \\ q^{2n} - q^{2n} - q^{2n-4} + q^{2n-4} \\ \vdots \\ q^{-2n+2} - q^{-2n+2} - q^{-2n-2} + i q^{2n} \\ q^{-2n} - i q^{2n+2} - i q^{2n-2} + i q^{2n-2} \\ \vdots \\ i q^{-2n+2} - i q^{-2n+2} - i q^{-2n-2} - q^{2n} \end{bmatrix} = 0, \quad \text{because}$$

most of these (ie all the $\vdots$) obviously go to zero; the non-obvious ones reduce to

$$\left.\begin{aligned} q^{2n+2} &= -i q^{-2n} \\ q^{-2n-2} &= i q^{2n} \\ i q^{2n+2} &= q^{-2n} \\ -i q^{-2n-2} &= q^{2n} \end{aligned}\right\} \iff q^{4n+2} = -i \quad \text{ie (3)}$$

$\square$

So,



(to see the last step, rotate head 180°)

$\square$