4·3·08

D(2n) Knot Inv. paper outline

1 everything you ever wanted to know about the D(2n) planar algebra but were afraid to ask

Def'n/Thm The D(2n) PA is gen (as a general PA) ~~by~~ by S, w/ rel'ns

1) $\delta = 2\cos(\quad)$
2) $JW(4n-3) = 0$
3) $\rho(S) = (-1)^{\cdots} S$
4) $\hat{S} = \hat{S} = 0$
5) $S \, S = \alpha \cdot \boxed{JW}$

~~B~~ this PA is pos-def, has p.graph D(2n)

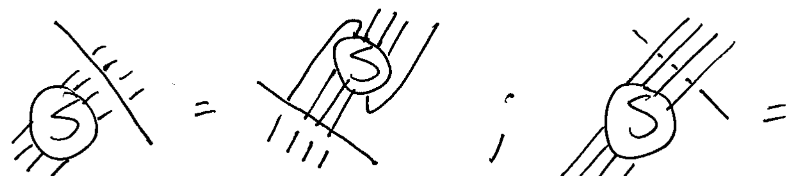
Easy consequences

~~Remarks~~:

- $S^{\supset} = S\bar{\supset} = S_{\supset} = \cdots = 0$
- $S^2 = JW(\quad)$
- higher JW? any time you have $4n-3$ strings, replace w/ Σ diagrams w/ fewer thru strings

Def'n: $\diagup\!\!\diagdown = ?_1 \, \underset{\frown}{\smile} + ?_2 \,)(\;;\; \diagdown\!\!\diagup = ?_2 \, \underset{\frown}{\smile} + ?_1 \, \underset{\frown}{\smile}$

Thm:



~~Thm: Basis of D(2n) PA is diagrams trees w/ a waist of $\leq 4n-4$ strands~~ diagrams? trees?

~~Pos~~:

~~Thm: Pos. defness~~

Lemma: one S.

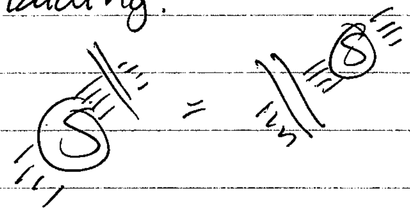
Projections.
$\sim$ of proj.
$\otimes$ of proj.
decomp. of proj.
bases for hom. spaces

Def: Principal graph

Compute p. graph of $D(2n)$ PA
Note: full $\otimes$ rules for $D(2n)$
Thm: Tree basis is basis
Def: $*$ (note: $S = S^*$)
Cor: pos. definiteness.

Note: ~~...~~ sub PA $A(4n-3) \subseteq D(2n)$
(projections are ...
$\otimes$ prod. rules ...) } 1 paragraph

$\boxed{\S 2}$ Braiding:



$\Rightarrow$ $\otimes$ cat of even projections is a braided $\otimes$ cat

apply standard procedure.
get knot invariant.

~~Note:~~
~~Con: $J(k, 4n-3-k)(q(\delta)) = J(4n-3-k\,?)(q(\delta))$~~
~~$J(\ldots)(q(\delta)) = J(P)(q(\delta)) + J(Q)(q(\delta))$~~
~~$D(2n)(\text{knot}) = \frac{1}{2} J(\ldots)(\text{knot})$~~

~~refined link invariant: $\ldots C = \ldots C$-subsets of # components~~

knots:

$D(2n)(\text{knot}) = \frac{1}{2}J(\sim)\ \text{knot}$

links:

2-component links:

$D(2n)(q(\sigma)) - J(n)(q(\sigma)) + \text{new}$

n-comp. links:

$C = \{2c\text{-subsets of components}\}$

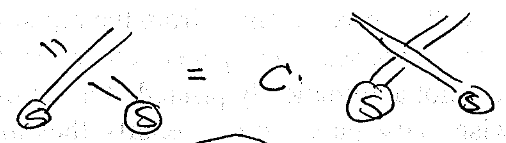
$\sum_{C}$ cabled knot w/ ⓢ inserted on each C

is an invariant

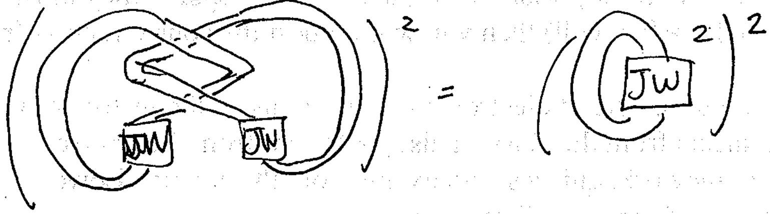
$\boxed{\S 3}$ Coincidences

- $D(4) \equiv 1$ on all links
- two refined pieces of $D(4) \equiv$ linking #s mod 3

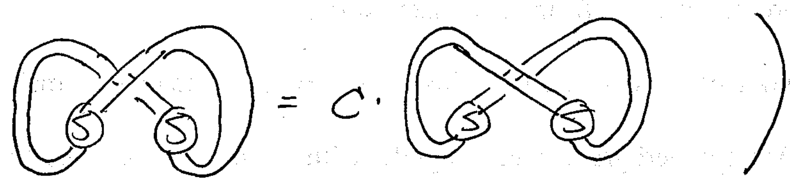
(proof: WTS



via



then



)

- link homotopy invariance
  (via quantum dimension 1)
  ~~(which of Milnor's?)~~

D(6) quantum dim of $P = \tau$

D(6)-inv $\equiv J($q that gives dim $\tau)$

If: rep. is self-dual, has 1 new component in $\otimes^2$

note: $\frac{1}{2}$ colored Jones on knots

$\equiv J($q that gives dim $\tau)$

D(8) $P \otimes Q = 1 \oplus X$

$P \otimes P = Y \oplus Z$

$\therefore$ D(8)-inv = specialization of HOMFLY

δ tells us ~~both~~ how to specialize one variable?

and of su(3)

D(10) don't expect D(10) & up to be specializations of HOMFLY, since

$P \otimes P = X \oplus Y \oplus Z$