'How to compute link invariants coming from the D_4 subfactor.' 1456

- A recipe.

  (no subfactors, just planar algebras; no braided tensor categories either!)

* Define the D_4 planar algebra.

- $d = \sqrt{3}$
- singly generated by $\boxed{R}$, positive!

* Consequences

- $f^{(5)} = 0$
- $\boxed{R}\ \boxed{R} \in TL$ (The planar algebra is $\mathbb{Z}_2$-graded by the number of R's)
- idempotents

* The recipe —

cable your link, insert idempotents

(observations about knots being boring..?)

- Why did that work?

  (somewhere in here perhaps mention a direct construction of the D_4 subfactor ($M \rtimes \mathbb{Z}_3$), and why that must be what we're talking about.)

. We 'expect' link invariants to come from braided tensor categories.

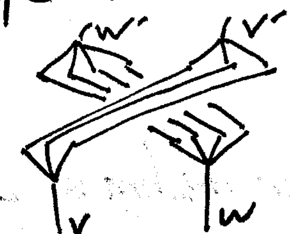
. That's almost the case here.

. Subfactors give tensor categories (almost!) and this tensor category is essentially the same thing as the planar algebra. (It's also the tensor category of bimodules.) generated by ${}_N M_M$.

— explain this, via the Karoubi envelope.

. But we don't expect these tensor categories to be braided.

- However, there is always a braiding on tensor powers of the generating bimodule!
  This is how the Jones polynomial was discovered
  $$Br_n \to TL_n \longrightarrow End(V^{\otimes n})$$
- This doesn't tell you how to braid everything, but because every bimodule lives inside some $V^{\otimes k}$, you can hope to define braidings using naturality:

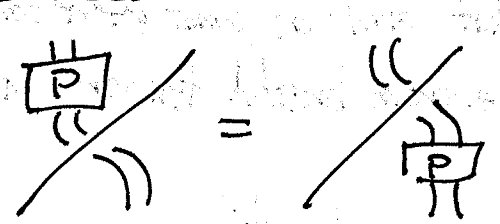
$b_{v,w} =$ 

$: V \otimes W \to W' \otimes V'$

(err... explain primes!)

In fact, if the principal graph starts off as A_3, there's only one possible braiding on $V \otimes \bar{V}$, and hence only one possible braiding on the whole category.

- Back to D_4.
  We need the relations



(and similarly with $Q = \frac{1}{2}(f_2 - R)$)

and, behold, that's the relation amongst the annular consequences!

Extras: D_{2n}, E_6, E_8, Haagerup!

- honest braidings require relations amongst annular consequences, so $d<2$.