The extended Haagerup subfactor

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Shanks Workshop on Subfactors and Tensor Categories
http://euclid.uhn.edu/~eep/Shanks2010_1.pdf

Outline

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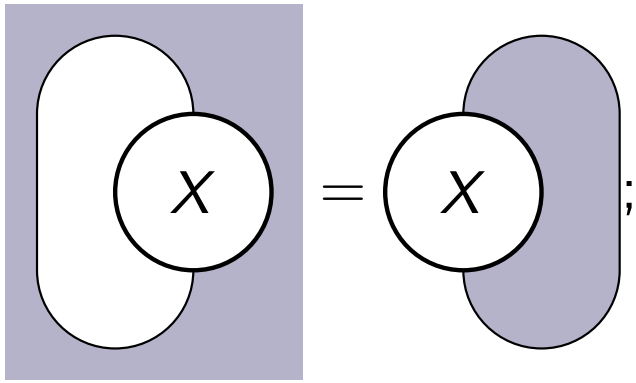
Subfactor planar algebras

A *subfactor planar algebra* has

- dim $V_{0,+} = \dim V_{0,-} = 1$;
- spherical trace: 

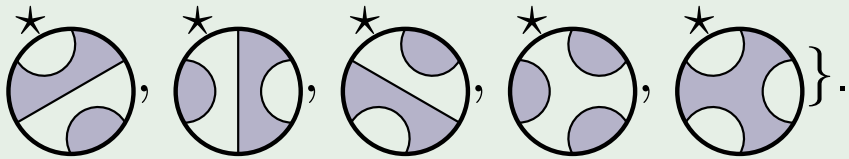
;
- an involution * on $V_{n,\pm}$, such that $\langle x, y \rangle = \mathrm{tr}(y^* x)$ is positive definite.

From these properties, it follows that closed circles count for a multiplicative constant δ.

Temperley-Lieb

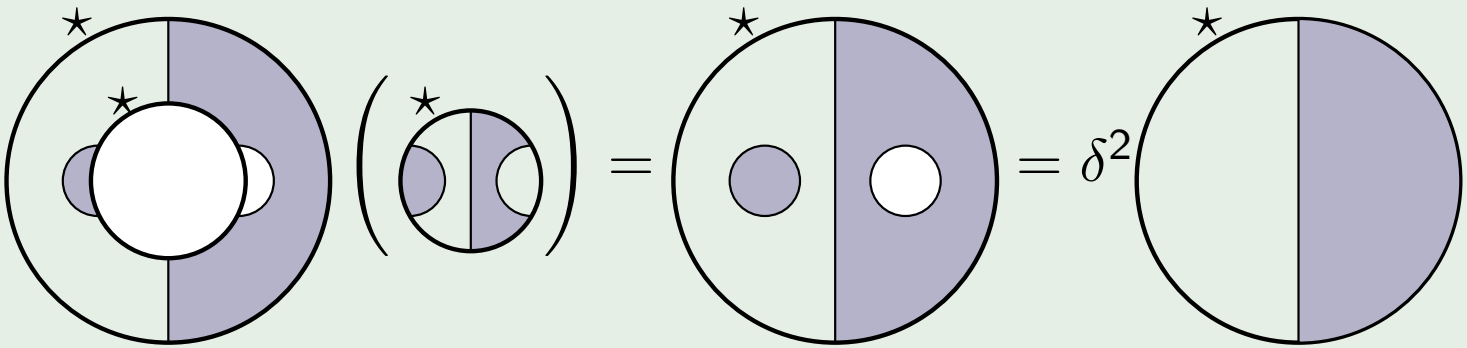
$TL_{n,\pm}(\delta)$ is the span (over $\mathbb{C}$) of non-crossing pairings of $2n$ points arranged around a circle, with formal addition.

Example

$TL_3 = \mathrm{Span}_{\mathbb{C}}\{$  $\}$.

Planar tangles act on TL by inserting diagrams into empty disks, smoothing strings,and throwing out closed loops at a cost of $\cdot\delta$.

Example



Invariants of subfactor planar algebras

Index

The index of a subfactor planar algebra δ^2 (δ is the value of a loop).

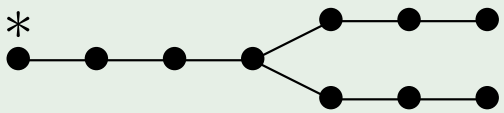
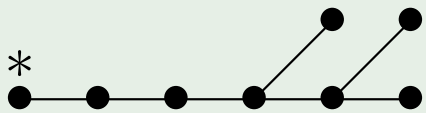
Principal graphs

The principal graphs of a subfactor planar algebra encode the fusion ($\otimes$) rules of its irreducible idempotents.

Note that the graph norm of the principal graphs is equal to δ!

Example

The Haagerup principal graphs

$H =$  , $H' =$ 

Which graphs are principal graphs?

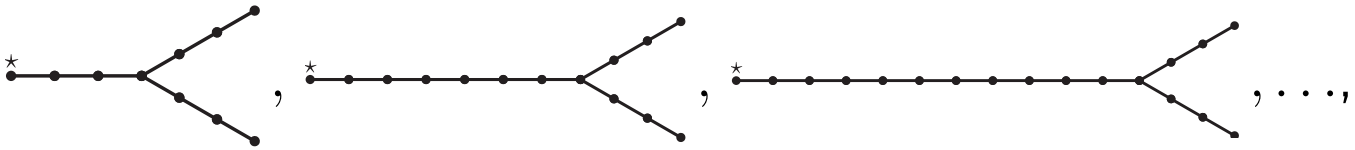
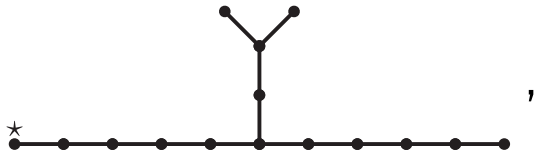
Subfactor planar algebras with index less than or equal to 4 have Dynkin diagrams or extended Dynkin diagrams as principal graphs.

Question

What are principal graphs for (finite-depth) subfactors with index slightly more than 4?

Haagerup (1994) found two families of candidates and one additional candidate, having index between 4 and $3 + \sqrt{3}$.

Classification of small-index subfactors

- Haagerup's possible principal graphs for subfactors with index less than $3 + \sqrt{3}$:
  - 
  - 
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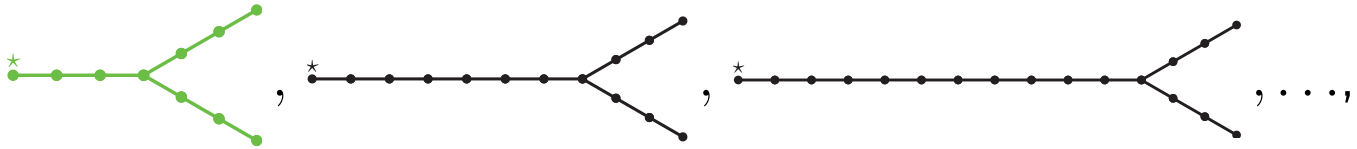
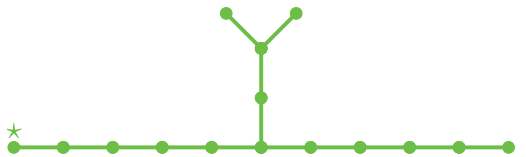
  - 
  - 
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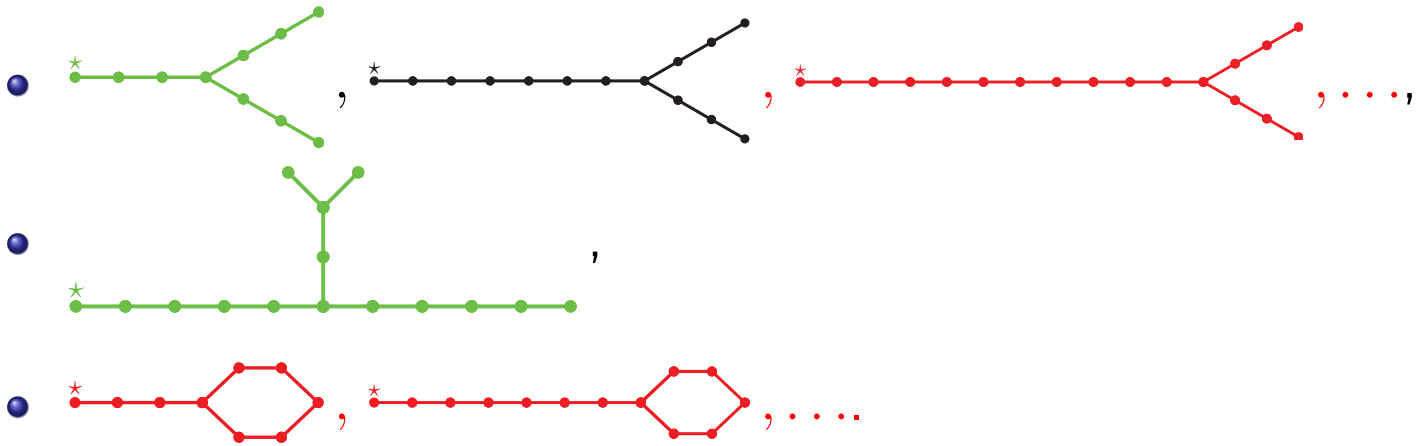

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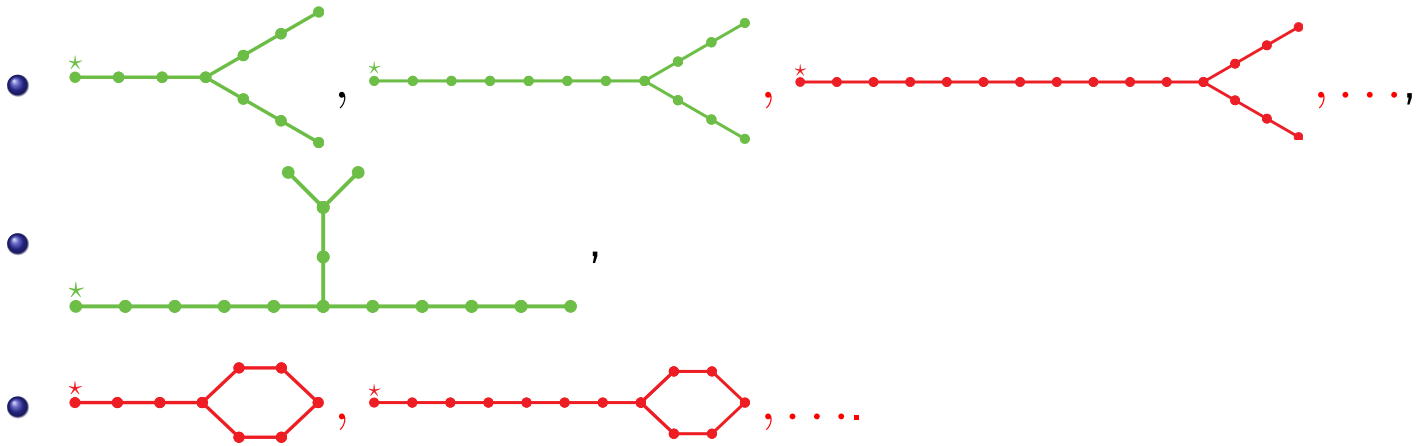

- Haagerup and Asaeda & Haagerup (1999) constructed two of these possibilities.
- Bisch (1998) and Asaeda & Yasuda (2007) ruled out infinite families.
- Today, we construct the missing example ('extended Haagerup'), and complete the classification.

Outline

And now for something completely different ...

The internal structure of the Haagerup and extended Haagerup planar algebras.

Emily

Generators and relations for the Haagerup family planar algebras

Stephen

Evaluation algorithm and basis for the Haagerup family planar algebras

Noah

Non-cyclotomicity of the Haagerup family planar algebras