

The principal graph determines the dimensions of objects and the dimensions of invariant spaces.

Example

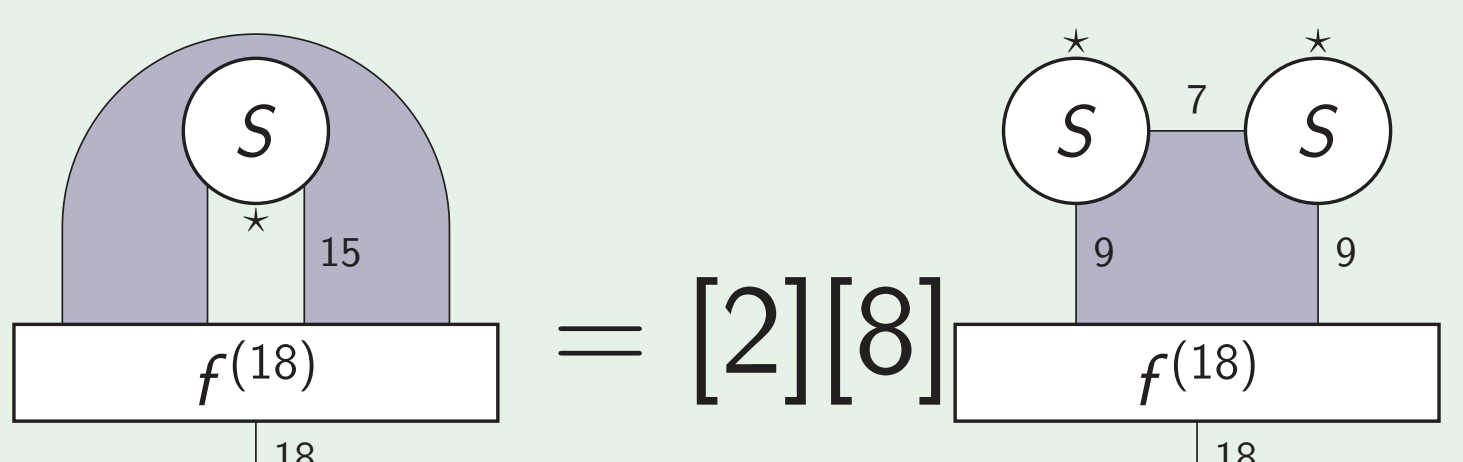
If $\Gamma = \underbrace{\text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---}}_{n \text{ edges}} \begin{matrix} \nearrow \dots \\ \searrow \dots \end{matrix}$, then

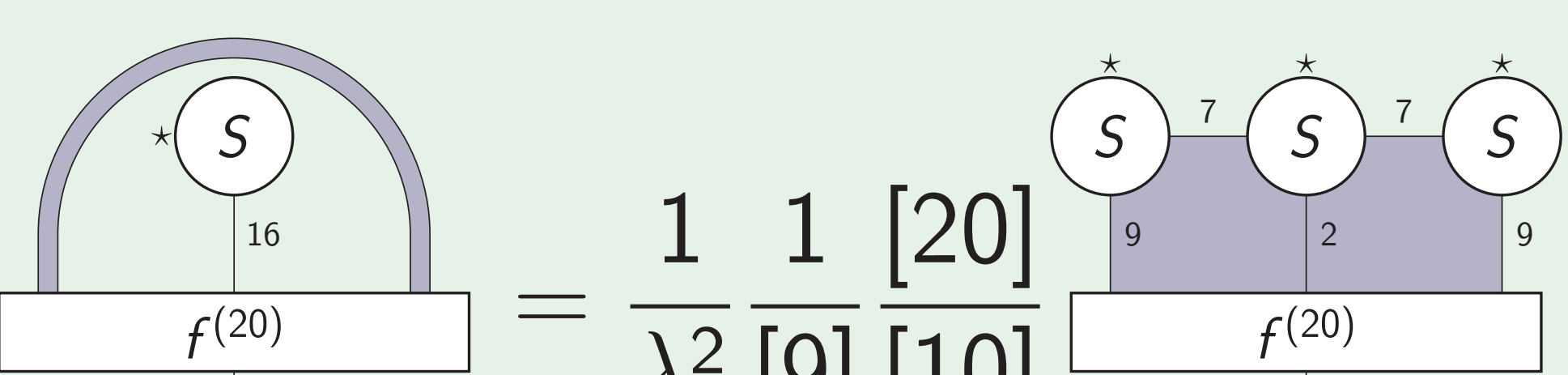
$$\dim \text{Inv}(V^{\otimes 2k}) = \begin{cases} C_k & \text{for } k \leq n \\ C_k + 1 & \text{for } k = n + 1. \end{cases}$$

These data strongly constrain generators and relations for the representation theory.

If a subfactor with principal graph $\text{---} \cdot \text{---} \cdot \text{---} \cdot \begin{matrix} \nearrow \\ \searrow \end{matrix}$ exists, its representation theory is generated by an 8-box S , which is a lowest weight vector with eigenvalue -1 and relations

1 $S^2 = \lambda^2 f^{(8)}$ (here $\lambda^2 = -\frac{1}{5} + 2 \text{Re} \sqrt[3]{\frac{117-65i\sqrt{3}}{2250}}$),

2 

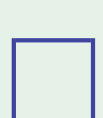
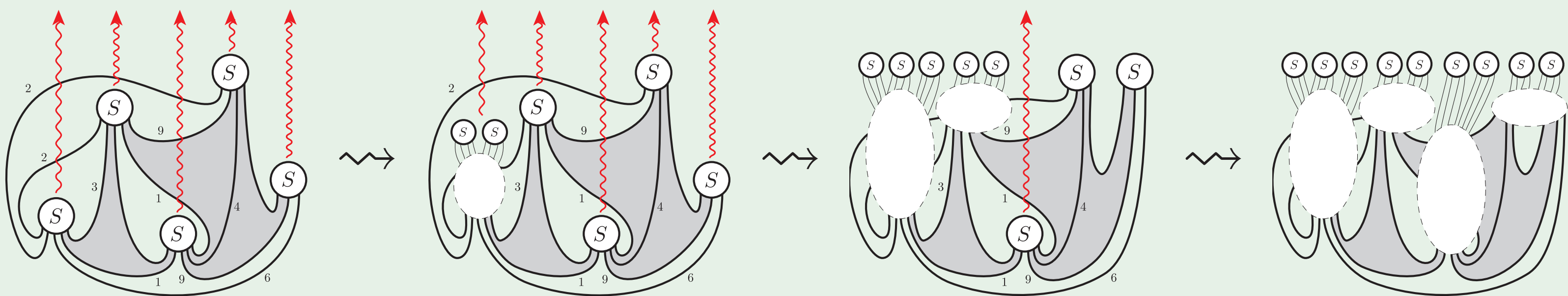
3 

Theorem (Bigelow-Morrison-Peters-Snyder, Acta Math. '09)

There is at most one subfactor with these generators and relations.

Proof.

These relations suffice to evaluate any closed diagram via the "jellyfish algorithm".



Two things could go wrong:

- The relations are inconsistent (i.e. this is a presentation of the 'zero subfactor').
- The resulting representation theory is not unitary.

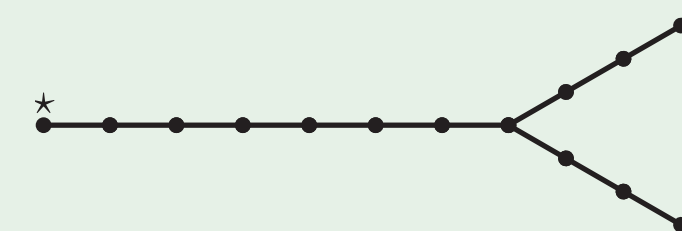
How do we check that relations are consistent? Every finite group sits inside some S_n . Analogously, we have

Theorem (Jones-Penneys arXiv:1007.317, Morrison-Walker)

Every subfactor planar algebra embeds in the graph planar algebra of its principal graph.

Theorem (BMPS '09)

There is an element of the GPA satisfying the desired relations. Unitarity is inherited from the GPA, so the subfactor

 *exists!*