Definition

The principal graph for a subfactor $N \subset M$ has vertices for the $N - N$, $N - M$, $M - N$ and $M - M$ bimodules, and an edge between Y and Z for each copy of Z appearing inside $Y \otimes X$. (Here $X = {}_N M_M$ or ${}_M M_N$ as appropriate.)

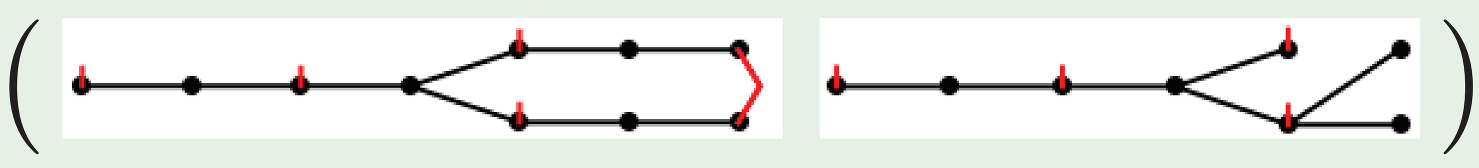
The principal graph has two connected components, the left N-modules and the left M-modules.

The graph norm is equal to the square root of the index of the subfactor (at least when the subfactor is finite depth, or is amenable).

Graph norm increases under inclusions.

We also remember which bimodules are dual to each other.

Example (The Haagerup subfactor's principal graph)



The principal graph must satisfy an associativity test:

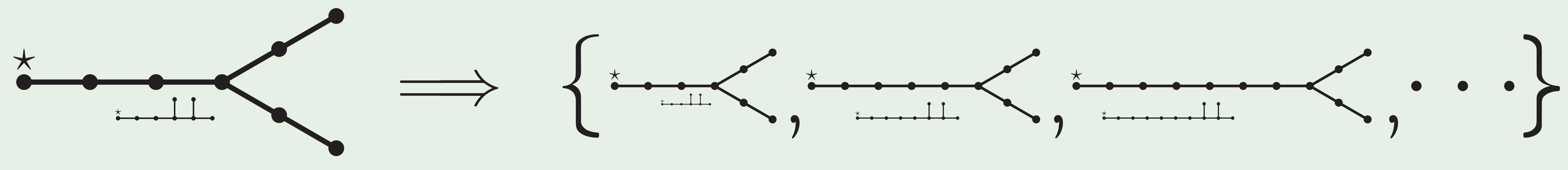
$$(X \otimes Y) \otimes X \cong X \otimes (Y \otimes X)$$

We can efficiently enumerate such pairs of graphs with index below some number L up to any rank or depth, obtaining a collection of allowed <u>vines</u> and <u>weeds</u>.

Definition

A vine represents an integer family of principal graphs, obtained by <u>translating</u> the vine.

Example



Definition

A weed represents an infinite family, obtained by either translating or <u>extending</u> arbitrarily on the right.

Example