Definition

A dimension function on a fusion category is a homomorphism from the Grothendieck ring to $\mathbb{C}$.

Definition

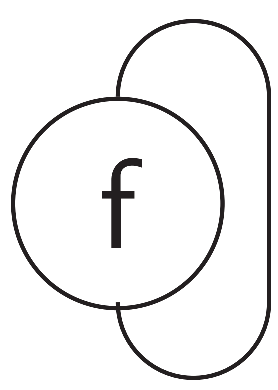
The principal graph for an object $X \in \mathcal{C}$ has vertices $Obj(\mathcal{C})$, and an edge from Y to Z for each summand of Z in $Y \otimes X$.

Definition

The Frobenius-Perron dimension of X is the largest eigenvalue of the principal graph for X.

The Frobenius-Perron dimension is always an algebraic integer.

In a $\otimes$-category with duals, endomorphisms have a trace:

$\mathrm{tr}(f) =$ 

$$= p_{X^*} \circ (f \otimes \mathbf{1}_{X^*}) \circ c_X$$

where $p_{X^*} : X \otimes X^* \to \mathbf{1}$ is the duality pairing, and $c_X : \mathbf{1} \to X \otimes X^*$ is the copairing.

Definition

The categorical dimension of an object is $tr(\mathbf{1}_X)$.

If the fusion category is unitary (there's a $*$-structure on Hom spaces, so $\mathrm{tr}(y^* x)$ is positive definite), then the Frobenius-Perron and categorical dimensions agree.