

## Example (Quantum $SU(2)$ at a root of unity)

The irreps of  $SU(2)$  (or  $U_q(\mathfrak{sl}_2)$ ) are indexed by natural numbers (the 'highest weight', which is one less than the dimension), and the tensor product rule is

$$V_a \otimes V_b = \bigoplus_{c=|a-b|}^{a+b} V_c.$$

When  $q$  is a root of unity, the tensor category has a 'negligible ideal'. We need to quotient by this to get a semisimple category. Only finitely many objects survive; when  $q$  is a  $4n - 4$ th root of unity, it's the first  $n$  objects. The 'truncated tensor product' is

$$V_a \otimes V_b = \bigoplus_{c=|a-b|}^{\min\{a+b, 2n-1-a-b\}} V_c.$$

Representations of finite groups always form a symmetric tensor category; representations of quantum groups form a braided tensor category. Both of these are special situations, however.

## Example

The category of  $G$ -graded vector spaces, with  $G$  a finite group, is neither symmetric nor braided when  $G$  is nonabelian.