Theorem (Calegari-Morrison-Snyder, CMP '10)

In any vine, only finitely many graphs have a cyclotomic index. With much better bounds, all but finitely many graphs have a multiplicity free eigenvalue which is not cyclotomic.

Either condition is sufficient to eliminate a possible subfactor.

- Penneys-Tenner `arXiv:1010.3797` have recently developed algorithms for efficiently computing these bounds,
- and computed them for the 43 vines in our enumeration.
- They looked at the finitely many cases remaining from the vines, and found obstructions for all but one graph.