Morrism

§1. Functorial knot invariants

A tangle invariant is just

$$F : \{\text{tangles}/\text{isotopy}\} \longrightarrow \mathcal{A}$$

Very naively a categorical invariant

$$\tilde{F} : \{\text{tangles}\} \longrightarrow \mathcal{C}$$

s.t. $T_1 \underset{\text{isotopic}}{\simeq} T_2 \Rightarrow \tilde{F}(T_1) \underset{\substack{\text{isom.} \\ \text{in } \mathcal{C}}}{\cong} \tilde{F}(T_2)$

This categorifies F if $K(\mathcal{C}) \cong \mathcal{A}$

$$[\tilde{F}(T)] = F(T)$$

We should be asking for

$$\tilde{F} : \{\text{tangles}\} \circlearrowleft \text{isotopies}/\text{isotopy} \longrightarrow \mathcal{C}$$

Khovanov homology doesn't quite look like this

① It requires tangle diagrams

$$\{\text{tangles}\} \circlearrowleft \cancel{\text{isotopies}}\ \text{cobordism} /\sim$$

$$\text{choosing plane proj.} \downarrow \quad \uparrow \text{inclusion}$$

$$\{\text{tangle diagrams}\} \circlearrowleft \text{movies}/\text{movie moves} \xrightarrow{Kh} \left\{ \begin{array}{l} \text{complexes of } H_n\text{-bimodule} \\ \text{or of diagrams in BN's setup} \end{array} \right\} \circlearrowleft \text{chain } \cancel{\text{equiv.}}\ \text{map} / \pm$$

sign problem

Kh/BN/Jacobsson

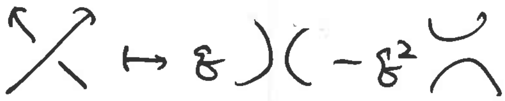
Surprisingly

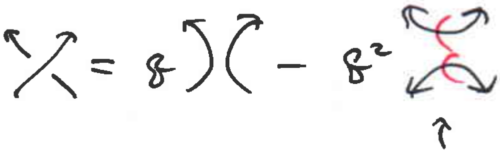
Kh gives much more

§2. Disorientations   M + Walker

categorify quantum $sl(2)$ stein theory instead of Kauffman

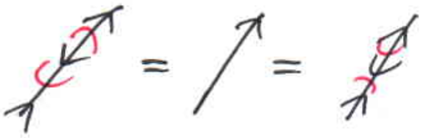
Kauffman (+writhe normalization)   quantum $sl(2)$

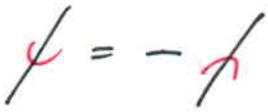




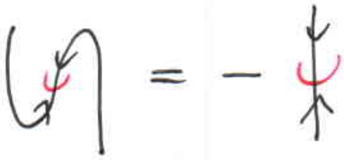
disorientation mark

$$O = q + q^{-1}$$





$: V \to V^*$



categorify this

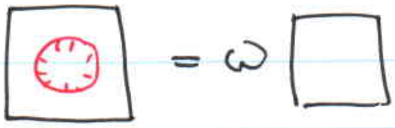
$DisCob(sl_2)$

Objects : disoriented flat tangles

morphism :

impose relations ① the usual oriented BN rel.

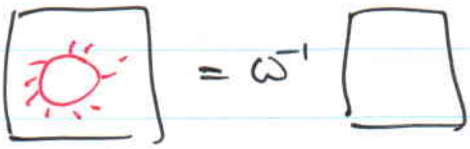
②

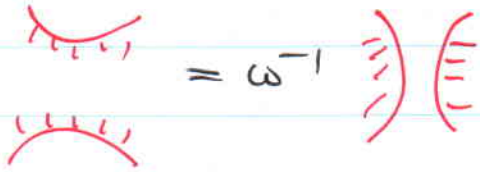


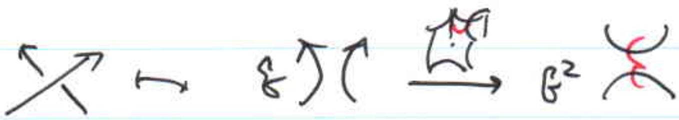
$\omega^4 = 1$

$\omega^2 = -1$

$\omega = 1$ recover the usual unori. theory

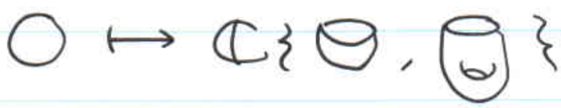




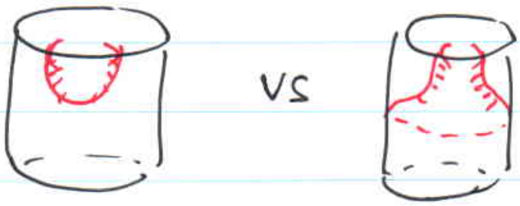


usually one translates BN $\rightsquigarrow$ Kh

by apply $\mathrm{Hom}(\phi, -)$.



The different circles are all 2-dim., but not canonically isomorphic

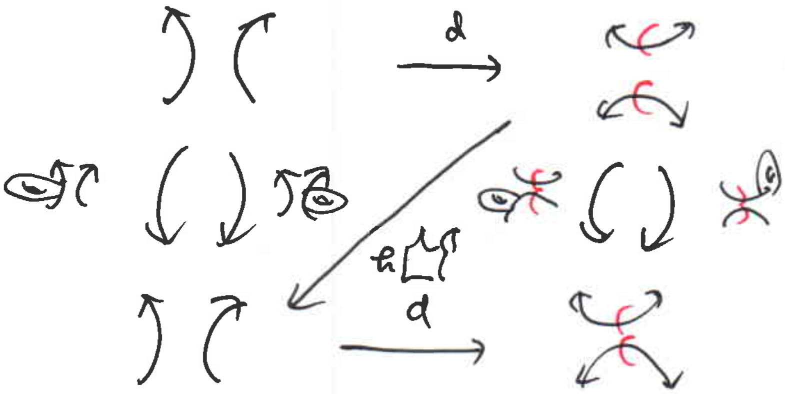


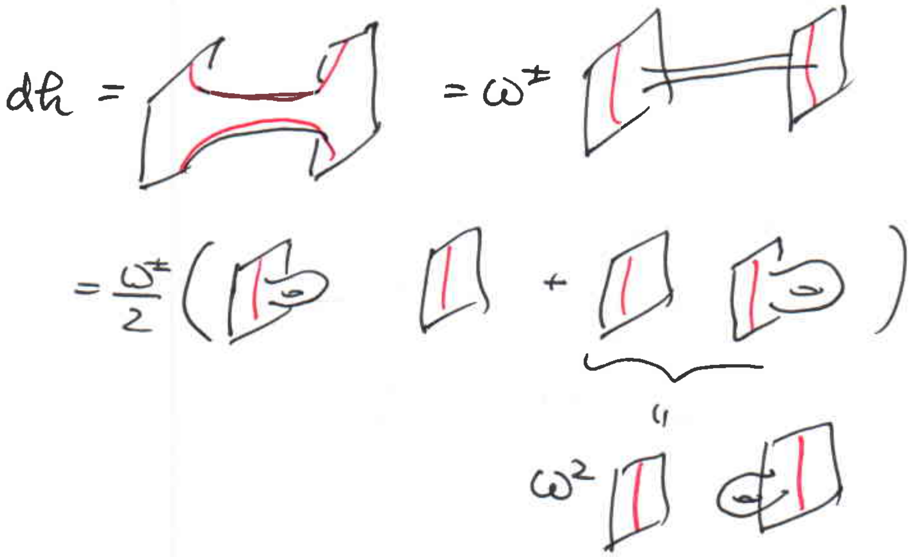
sign is different.

We also have to write down chain maps
for each Reidemeister moves.

tedious!

math.GT/0701339



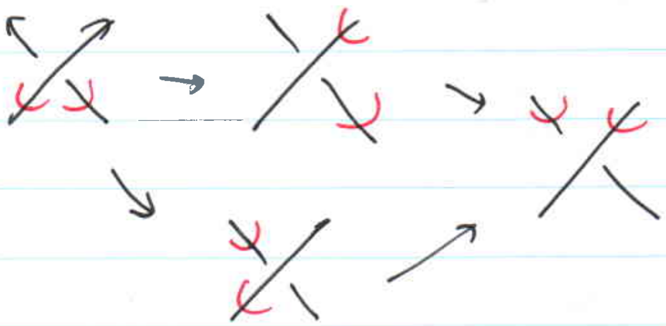


Extension to disoriented tangles

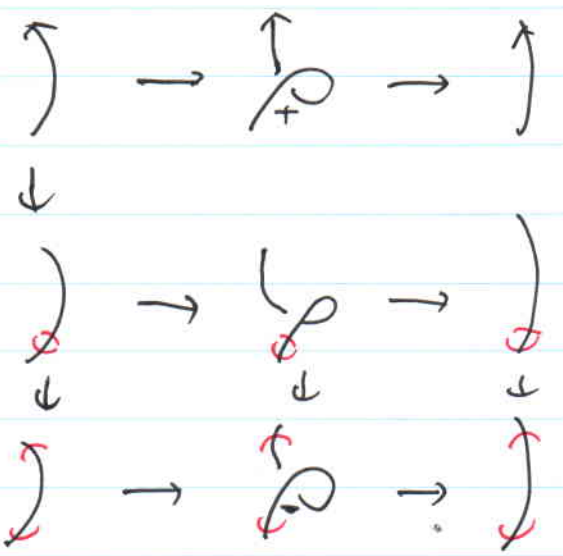
- There are more Reidemeister moves

Chain map does not preserve height

There are more movie moves



RI can follow from other movie moves :

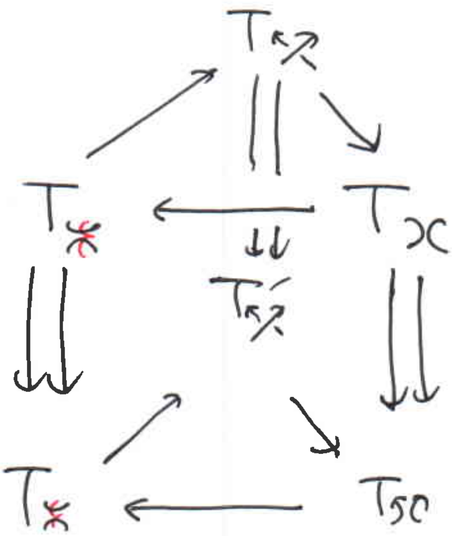


⇐ not quite this example

- We can associate chain maps to nonorientable surfaces

gives a generator in $Kh(3_1)$

but the Möbius band gives 0 in $Kh(\overline{3}_1)$



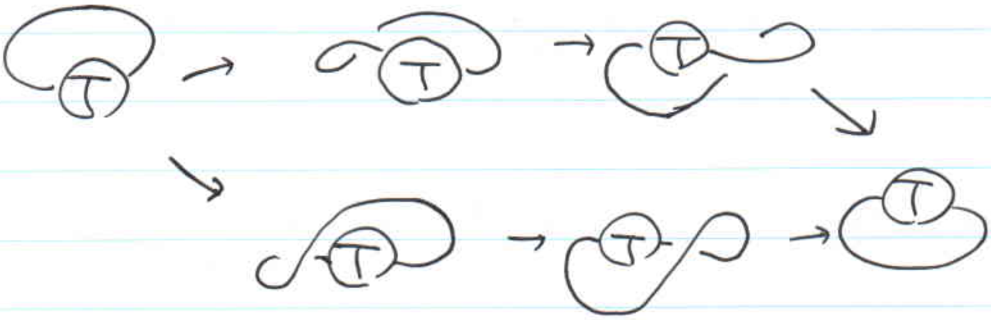
2 morphisms

Kh is functorial for knots in S^3, not just B^3

Cobordisms in S^3 generically miss ∞

but isotopies of cobordisms don't

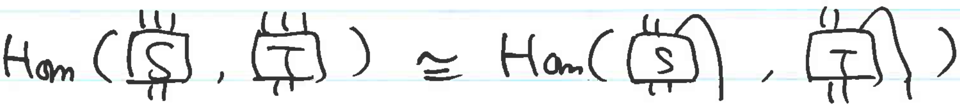
∃ extra movie move in S^3



duality

tangles S, T, U

$$\mathrm{Hom}(\boxed{S}, \boxed{T}\boxed{U}) \cong \mathrm{Hom}(\boxed{S}\boxed{\bar{U}}, \boxed{T})$$



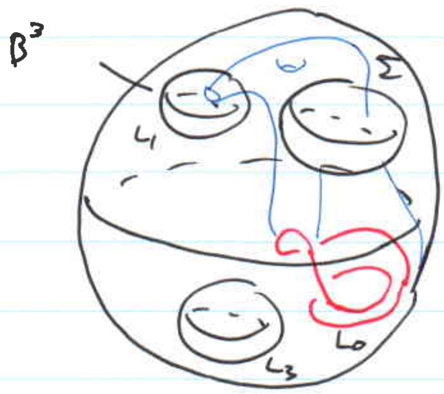
There is a good notion of braided tensor 2-categories with duals at 2-levels (LSR, BL)

These Isomorphisms also exist at level of chain maps between homologies

but tedius

" Khovanov homology gives an algebra over the lasagna operad"

B^3



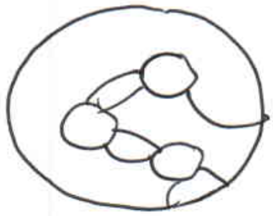
→ ∃ depending only on Σ up to isotopy

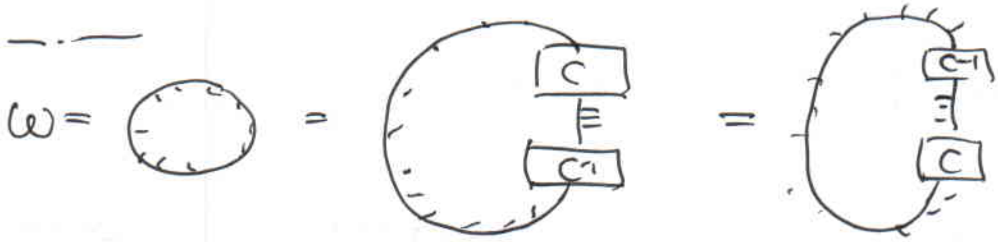
$$\otimes Kh(L_i) \longrightarrow Kh(L_0)$$

These picture form an operad,

These maps on Kh respect that structure.

A planar algebra is an algebra over the spaghetti & meat balls operad





spin framing