

What is a fusion category?

"the simplest model for a system of particles of different types, that can merge and split"

* the category of representations of a fixed finite group.

Definition a tensor category with duals that's unitary and spherical and has finitely many simple objects.

Duals 'every particle has antiparticle' $*$: $a \mapsto a^*$
 'fusion channels' 'pair creation' 'pair annihilation'
 $\cup: 1 \rightarrow a^* \otimes a$ $\cap: a^* \otimes a \rightarrow 1$

Unitary need an antilinear involution $\text{Hom}(a, b) \rightarrow \text{Hom}(b, a)$
 so the inner product $\langle f, g \rangle = \text{tr}(f, g)$ is positive definite.

Spherical $\text{tr}(f) = \text{tr}(f) = \text{tr}(f)$
 $a \circ (1 \otimes f) \circ c$
 $c \cdot 1 \rightarrow a \otimes a^* \xrightarrow{\text{id} \otimes f} a \otimes a^* \rightarrow 1$
 $c \cdot 1 \rightarrow a^* \otimes a \xrightarrow{f \otimes \text{id}} a^* \otimes a \xrightarrow{\circ} 1$