**AUSTRALIAN RESEARCH COUNCIL**
**Discovery - Projects**
**Proposal for Funding Commencing in 2014**

**DP**

**PROJECT ID: DP140100732**

**First Investigator: Dr Scott Morrison**

**Admin Org: The Australian National University**

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|---|
| Total number of sheets contained in this Proposal: 61 |

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## CERTIFICATION

### Certification by the Deputy/Pro Vice-Chancellor (Research) or their delegate or equivalent in the Administering Organisation

I certify that—

- I have obtained the agreement, attested to by written evidence, of all the relevant persons and organisations necessary to allow the Project to proceed. This written evidence has been retained and will be provided to the ARC if requested.
- I have read, understood and complied with the *Discovery Projects Funding Rules for funding commencing in 2014* and, if the Proposal is successful, I agree to abide by the terms and conditions of the *Discovery Projects Funding Agreement for funding commencing in 2014*.
- Proper enquiries have been made and I am satisfied that the participants and the organisations listed in this Proposal meet the requirements specified in the *Discovery Projects Funding Rules for funding commencing in 2014.* I will notify the ARC if there are changes to any named Participant or organisation after the submission of this Proposal.
- This organisation supports this Proposal and if successful will provide basic facilities and the items listed in the budget for the Project.
- To the best of my knowledge, all Conflicts of Interest relating to parties involved in or associated with this Proposal have been disclosed to the ARC, and I will notify the ARC of any changes which arise after the submission of this Proposal.
- This Proposal is not substantially aimed at understanding or treating a human disease or health condition. The definition of Medical and Dental Research is located on the ARC website.
- This Proposal does not duplicate Commonwealth-funded research including that in a Commonwealth-funded Research Centre.
- I consent, on behalf of all the parties, to this Proposal being referred to third parties, who will remain anonymous, for assessment purposes.
- I consent on behalf of all the parties, to the ARC copying, modifying and otherwise dealing with information contained in the Proposal.
- To the best of my knowledge, the Privacy Notice appearing at the top of this Proposal form has been drawn to the attention of all the Participants whose personal details have been provided at the Personnel section.
- To the best of my knowledge all details provided in this Proposal form and in any supporting documentation are true and complete in accordance with Section 14.5 of the *Discovery Projects Funding Rules for funding commencing in 2014*.
- If this Proposal is successful, I am prepared to have the Project carried out at this organisation as set out in this Proposal and in accordance with the *Discovery Projects Funding Rules for funding commencing in 2014*.
- The Project can be accommodated within the general facilities in this organisation, and if applicable, within the facilities of other relevant organisations specified in this Proposal, and sufficient working and office space is available for any proposed additional staff.
- All funds for this Project will only be spent for the purpose for which they are provided.
- The Project will not be permitted to proceed until appropriate ethical clearance(s) has/have been obtained.
- I understand and agree that all statutory requirements must be met before the Project can commence.

## PART A - Administrative Summary (DP140100732)

### A1. If this proposal is successful, which organisation will it be administered by?

**Administering Organisation Name**

The Australian National University

### A2. Proposal Title

(Provide a short descriptive title of no more than 150 characters (20 words). Avoid the use of acronyms, quotation marks and upper case characters.)

Symmetries of subfactors

### A3. Person Participant Summary

| | Person number | Family name | First name | Current organisation |
|---|---|---|---|---|
| 1 | 1 | Morrison | Scott | The Australian National University |
| 2 | 2 | Grossman | Pinhas | The University of New South Wales |
| 3 | 3 | Jones | Vaughan | University of California, Berkeley, Vanderbilt University |

| | Relevant organisation for this proposal | Role |
|---|---|---|
| 1 | The Australian National University | Chief Investigator |
| 2 | The University of New South Wales | Chief Investigator |
| 3 | Vanderbilt University | Partner Investigator |

### A4. Organisation Participant Summary

| | Organisation number | Short name | Name | Role |
|---|---|---|---|---|
| 1 | 1 | ANU | The Australian National University | Administering Organisation |
| 2 | 2 | UNSW | The University of New South Wales | Other Eligible Organisation |
| 3 | 3 | Vanderbilt University | Vanderbilt University | Other Organisation |

### A5. Summary of Proposal

(In no more than 750 characters (approx 100 words) of plain language, summarise aims, significance and expected outcomes.)

A subfactor is a mathematical object that encodes "quantum" symmetries which may be thought of as generalizations of group symmetries. We will study subfactors and classify families of subfactor symmetries which include the exotic subfactors of small index. We will also develop computational tools for analyzing and cataloguing these symmetries. This project contributes to the development of operator algebra theory, and the new mathematical fields of quantum algebra and quantum topology; it also has applications to physical models.

### A6. Summary of Project for Public Release

(In no more than 350 characters (approx 50 words), please provide a two-sentence descriptor of the purpose and expected outcome of the project which is suitable for media or other publicity material. Do not duplicate or simply truncate the 'Summary of Proposal'.)

This project will involve mathematical research of the highest international calibre on subfactors. Progress in these fields will lead to advances in the mathematical fields of operator algebra, quantum algebra, and quantum topology, with applications to computing (e.g. substrates for quantum computers) and condensed matter physics.

## PART B - Classification and other statistical information (DP140100732)

### B1. National Research Priorities

**Select 'Yes' to indicate which of the National Research Priorities this Proposal falls within. Select 'No' if not applicable.**

(Refer to the ARC website for further information regarding National Research Priorities.)

**National Research Priority Selected**

| |
|---|
| Yes |

**National Research Priorities**

| | National Research Priority Area | National Research Priority Goal |
|---|---|---|
| 1 | Frontier Technologies for Building and Transforming Australian Industries | Breakthrough sciences |

### B2. Field of Research (FOR)

| | Field of Research (FOR) | Field of Research (FOR) Percent |
|---|---|---|
| 1 | 010108 - Operator Algebras and Functional Analysis | 50 |
| 2 | 010103 - Category Theory, K Theory, Homological Algebra | 40 |
| 3 | 010501 - Algebraic Structures in Mathematical Physics | 10 |

### B3. Socio-Economic Objective (SEO-08)

| | Socio Economic Objective (SEO) | Socio Economic Objective (SEO) Percent |
|---|---|---|
| 1 | 970101 - Expanding Knowledge in the Mathematical Sciences | 100 |

### B4. Keywords

| | Keywords |
|---|---|
| 1 | subfactors |
| 2 | operator algebras |
| 3 | tensor categories |

### B5. If the proposed research involves international collaboration, please specify the country/ies involved.

| | International Collaboration Country Name |
|---|---|
| 1 | United States of America |
| 2 | Japan |
| 3 | United Kingdom |

**C1. Please upload a Project Description as detailed in the Instructions to Applicants in no more than 10 A4 pages and in the required format.**

**Attached PDF**

## Project Title: "Symmetries of subfactors"

## Aims and background

This project aims to develop the theory of subfactors. Subfactors are mathematical structures which generalize groups by allowing a broader range of "quantum" symmetries than classical structures like groups.

*We will study the symmetries of subfactors, with a particular emphasis on identifying the combinatorial and group-like symmetries of the exotic subfactors with small index.* We will combine ideas from operator algebras, planar algebras, and the representation theory of fusion categories. The project will require developing new computational tools for classifying the combinatorial structures that arise from these symmetries. This project will build upon the existing research programs of the chief investigators; both have worked extensively on subfactors and fusion categories. Pinhas Grossman has worked intensely on the representation theory of fusion categories, and Scott Morrison has expertise in planar algebras. Vaughan Jones, as partner investigator, brings deep expertise in planar algebras, subfactors and operator algebras.

The primary goals of this project are:

- to construct the conjectural missing Asaeda-Haagerup subfactors,
- to generalize the near-group and Izumi-Haagerup series of subfactors,
- to construct new subfactors and fusion categories by solving more instances of these series,
- to find the Brauer-Picard groupoids of new exotic subfactors, included the extended Haagerup subfactor and the '4442' subfactor,
- to develop computational machinery to search for and classify fusion rings and groupoids,
- to find skein relations for the planar algebras of the exotic subfactors,
- to compute the tube algebras and the quantum doubles of the exotic subfactors, and
- to classify families of subfactors according to the symmetries of their invariants

*Each of these is a significant and important problem in the modern study of operator algebras or quantum algebra.* In the course of achieving these goals, we will:

- Develop new tools to construct examples of subfactors.
- Develop new tools to classify subfactors by controlling their combinatorial invariants.
- Develop, document and distribute computer research tools for the study of subfactors and fusion categories.
- Participate in international collaborations, and build connections between Australian mathematics and the international mathematical community.

**Background** Subfactors are a relatively new mathematical area, having been studied since the 1980s. Subfactors are particular examples of von Neumann algebras, and are studied for their symmetries, which have a rich algebraic and combinatorial structure. These symmetries can be thought of generalizations of groups, or "quantum" groups, and in this sense the study of subfactors forms part of the growing network of mathematical disciplines with a "quantum" flavor. Such disciplines include quantum groups, quantum algebra, quantum topology, algebraic quantum field theory, and quantum computing.

Algebras of operators on Hilbert space were introduced by von Neumann in the early twentieth century as a mathematical framework for quantum mechanics, uniting the matrix mechanics of Heidegger and the wave mechanics of Schrödinger. These von Neumann algebras replaced the commuting observables of classical mechanics with non-commuting linear operators on infinite-dimensional vector spaces. The non-commutativity of the position and momentum operators explains the impossibility of their simultaneous measurement, famously formulated in the Heisenberg uncertainty principle.

A subfactor is a pair of von Neumann algebras with trivial centres, along with an embedding of one inside the other. The object of interest here is not so much the individual von Neumann algebras but the embedding and its symmetries. A motivating example is obtained from an action of a finite group on a factor; the subfactor is the fixed points under the group action. Here one can recover the group from the subfactor. More generally, one thinks of a subfactor as a generalized group and subfactor theory as a noncommutative Galois theory.

**Definition 1.** *A von Neumann algebra is an algebra of operators on a Hilbert space which is the commutant of a unitary group representation. A subfactor is a unital inclusion $N \subseteq M$ of von Neumann algebras with trivial centers.*

The symmetries of a subfactor are described by a 'standard invariant' [Jon99, Ocn88, Pop95], which is a bimodule category between two tensor categories along with a distinguished object.

**Definition 2.** *A subfactor $N \subseteq M$ has finite index if $M$ is a finitely generated projective $N$-module. The standard invariant of the subfactor is the tensor category of $N-N$ bimodules tensor-generated by ${}_N M_N$ together with the module category of $N-M$ bimodules generated by ${}_N M_M$.*

This standard invariant can also be described as a planar algebra. A planar algebra is an algebra over the operad of planar tangles; namely it is a sequence of vector spaces on which planar diagrams act. Here the vector spaces which admit the planar action are endomorphism spaces of certain objects in the categories of $N-N$ and $N-M$ bimodules. In this way planar topology becomes a crucial tool in the study of subfactors.

The fundamental problem of subfactors is to classify them. What symmetries exist besides those coming from groups? A remarkably successful classification program exploits the notion of index for subfactors, which is a generalization of the notion of index for subgroups of a group. Unlike for groups, the index of a subfactor need not be an integer.

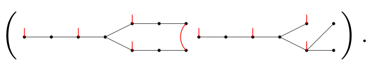
The ground-breaking theorem of Jones' says that this index is quantised:

**Theorem 1.** *[Jon83]: The index of a subfactor $N \subseteq M$ is either of the form $4\cos^2 \pi/n$ or is at least 4.*

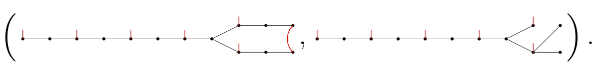
When first discovered this was an extraordinary surprise. We now have several related explanations: the theorem can be reduced to easier results about the norms of graphs, or about totally positive cyclotomic integers. The classification of subfactors achieving these index values below 4 was initiated by Ocneanu [Ocn88] and completed by a number of researchers over the following decade [GdlHJ89, Izu91, BN91, Izu94, Pop94, Kaw95, KO02]. The classification of subfactors with index above 4 was begun in the 1990s by Haagerup, who gave an exhaustive list possibilities up to index $3+\sqrt{3}$, but did not initially rule out all the cases or establish all the required uniqueness results. This classification was finally completed, and then extended to index 5, by Scott Morrison, Vaughan Jones and other collaborators.

**Theorem 2** ([M:4] [M:3] [M:2]/[J:2] [M:7] [M:10] [PT12, Haa94, AH99, GdlHJ89, Izu01, Han10])**.** *If an extremal subfactor has index between 4 and 5 its standard invariant is Temperley-Lieb or it is one of the following ten planar algebras.*

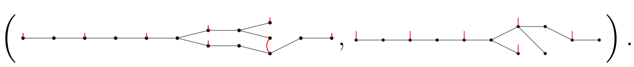
- *The Haagerup planar algebra [AH99] and its dual, with index $\frac{5+\sqrt{13}}{2}$ and principal bigraph pair*



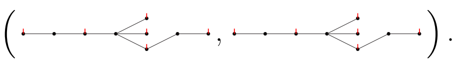
- *The extended Haagerup planar algebra [M:10], with index $\frac{8}{3} + \frac{2}{3} \operatorname{Re} \sqrt[3]{\frac{13}{2}\left(-5-3i\sqrt{3}\right)}$*



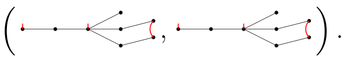
- *The Asaeda-Haagerup planar algebra [AH99] and its dual, with index $\frac{5+\sqrt{17}}{2}$*



- *The 3311 Goodman-de la Harpe-Jones planar algebra [GdlHJ89] and its dual, with index $3+\sqrt{3}$*



- *Izumi's self-dual 2221 planar algebra [Izu01] and its complex conjugate, with index* $\frac{5+\sqrt{21}}{2}$



Among the list of subfactors with indices between 4 and 5 are several examples coming from group actions and conformal field theory. But there are also several "exotic" examples which contain interesting new symmetries that are not recognized from any other area of mathematics or physics. These exotic examples include the Haagerup subfactor, the Asaeda-Haagerup subfactor, and the 'extended Haagerup' subfactor.

Understanding the symmetries of these exotic subfactors is an exciting mathematical problem. A major goal of this project is to place these examples in more general families of subfactors and construct new examples. Indeed, for the Haagerup subfactor such a generalized series is already conjectured to exist based on work of Izumi [Izu01], who constructed the Haagerup subfactor as an endomorphism of a Cuntz algebra corresponding to the group $\mathbb{Z}/3\mathbb{Z}$, and wrote down equations for arbitrary Abelian groups. Izumi then solved these equations for $\mathbb{Z}/5\mathbb{Z}$, and further solutions were obtained by Evans and Gannon [EG12].

A key element in understanding the symmetries of a subfactor is working out its representation theory in the sense of [ENO09]. This representation theory can be expressed as a groupoid of Morita equivalent fusion categories associated to the subfactor, called the Brauer-Picard groupoid.

Together with Noah Snyder, Pinhas Grossman has worked out much of the representation theory of the Haagerup and Asaeda-Haagerup subfactors [G:1] [GS12] and developed some general machinery for exploring similar subfactors [GJS12], which has led to the discovery of many new subfactors and fusion categories.

**Theorem 3.** *The Brauer-Picard groupoid of the Haagerup subfactor contains exactly three inequivalent fusion categories with only one bimodule category between each pair of fusion categories. The Brauer-Picard groupoid of the Asaeda-Haagerup subfactor contains at least three inequivalent fusion categories with exactly four bimodule categories between each pair of fusion categories.*

This result led to the conjecture that the Asaeda-Haagerup subfactor also belongs to a family of subfactors analogous to that for the Haagerup subfactor.

## Research project

**Approach** Our approach to the classification of subfactors and their symmetries will be a synthesis of several mathematical disciplines, including operator algebras, representation theory, tensor categories, and planar algebras. We will also develop algorithms to attack difficult combinatorial problems which arise in applying the general theory to specific examples.

The classification of subfactors has so far largely been focused on subfactors of small index. Several exotic subfactors of small index have been found, and their invariants extensively studied. The main goal of our project is to investigate the general classes of subfactors to which these exotic examples belong.

The basic idea is to analyze the structure of the known exotic subfactors using the representation theory of the associated tensor categories, while at the same time utilizing constructions from operator algebras and planar algebras to find new exotic subfactors, both in the same families as the known ones, and potentially entirely new ones. Combining the results of this two-pronged attack will yield, in addition to new examples of subfactors, a greater understanding of subfactor symmetries and context for the surprising appearance of the exotic subfactors.

We will now discuss in detail how this general approach will be applied to address the specific problems listed above in the Aims and Background section.

**The missing Asaeda-Haagerup subfactor.** There are several 'missing' Asaeda-Haagerup subfactors which we conjecture have even parts Morita equivalent to the even parts of the Asaeda-Haagerup subfactor. Strong evidence for the existence of such subfactors appeared in the analysis of the Brauer-Picard groupoid encoding the representation theory of the Asaeda-Haagerup subfactor [GS12]. This groupoid is known to contain at least three inequivalent fusion categories and symmetry group of automorphisms of each of the fusion categories in the groupoid is the Klein 4-group.

The existence of the missing Asaeda-Haagerup subfactors would also imply the existence of three additional fusion categories in the Brauer-Picard groupoid. Unlike the known Asaeda-Haagerup categories, these new categories would each contain 4-element groups of invertible objects (in fact, both $\mathbb{Z}/4\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ would appear, in different categories). The existence of these subfactors would thus reveal new group-like symmetries within the Asaeda-Haagerup subfactor analogous to those that that are already known for the Haagerup subfactor.

There are several available approaches to constructing these subfactors, including connections, planar algebras, and endomorphisms of Cuntz algebras. We will immediately begin working on several of these approaches in parallel, in collaboration with Noah Snyder, and we believe we will be able to construct these missing subfactors.

**Problem 1.** *Construct the missing Asaeda-Haagerup subfactors.*

**Operator algebraic constructions of group-like subfactors.** A general method for constructing subfactors from endomorphisms of the Cuntz C*-algebras was introduced by Izumi in [Izu93, Izu98] and further developed by the same author in [Izu00, Izu01]. In this construction, the C*-algebra is considered as a category whose objects are finite-index endomorphisms and whose morphisms are elements of the algebra which intertwine these endomorphisms.

The Cuntz algebra construction has been used to recover the Haagerup subfactor and construct various generalizations [Izu01, EG11], as well to construct and classify certain near-group fusion categories [EG12], which are categories with only one non-invertible object. The construction involves two distinct steps: finding a set of equations for structure constants associated to a group; and then finding solutions to these equations for specific groups.

In this project we will develop general procedures for both parts of the construction. We will begin by studying the known equations for the Izumi-Haagerup family and the near-group family of subfactors. Here we will develop computer algorithms to solve these equations for larger groups, and then analyze patterns in these solutions to find general families of solutions. We have already met with some success in this approach, having solved Izumi's equations for the group $\mathbb{Z}/8\mathbb{Z}$ and constructed new subfactors from the solution.

A key technique in our analysis will be to formulate the Cuntz algebra equations in the more algebraic framework of tensor categories (rather than the analytic framework of sectors), and then apply the algebraic machinery associated to such categories. This will allow us to generalize the equations to new familes of subfactors, leading to new examples of subfactors and fusion categories.

**Problem 2.** *Find solutions to the equations for the Izumi-Haagerup and near-group subfactors for larger groups. Obtain both sporadic solutions and infinite series of solutions.*

**Problem 3.** *Formulate equations for similar classes of subfactors associated to finite groups, and solve these equations.*

**Group-like symmetries of the exotic subfactors via representation theory.** The exotic Haagerup subfactor has a symmetry of $\mathbb{Z}/3\mathbb{Z}$ which was exploited by Izumi in his Cuntz algebra construction, thereby allowing the generalization to a family of subfactors associated to arbitrary Abelian groups (see [Izu01, EG11]). The construction of the missing Asaeda-Haagerup subfactor will reveal a similar structure, as discussed above.

In this project we will bring the remaining known exotic subfactors into a similar framework by identifying their group-like and combinatorial symmetries. Particular subfactors of interest here are the 4442 subfactor [M:17] and the extended Haagerup subfactor [M:10], both constructed by Scott Morrison and his collaborators. Our approach to this is twofold: we will compute the Brauer-Picard groupoids of these subfactors and identify all the subfactors in the Morita classes. This work will follow the general methods used by Pinhas Grossman and Noah Snyder in [G:1][GS12]; we expect to collaborate with Snyder on this portion of the project.

Computing the Brauer-Picard groupoid will involve developing algorithms to classify fusion rings and groupoids. We have already done a good deal of computational work in this area and we will develop and distribute software that will be useful beyong this specific project.

Once we have computed the Brauer-Picard groupoids, we will use planar algebras and connections to analyze the symmetries of the most tractable subfactors in these classes. Scott Morrison and Vaughan Jones are each leading experts on the skein theory of planar algebras in general and Scott Morrison has done considerable work on its applications to classification of exotic subfactors in particular.

**Problem 4.** *Develop algorithms to find fusion rings and classify fusion groupoids associated to those fusion rings.*

**Problem 5.** *Find the Brauer-Picard groupoids of exotic subfactors, including the 4442 subfactor and the extended Hagerup subfactor.*

**Skein theory of the exotic subfactors.** A novel and powerful approach to the construction and classification of subfactors was introduced by Vaughan Jones in his algebraic formalism of planar algebras [Jon99]. Here the idea as that one can express the invariants of subfactors as generators of planar diagrams satisfying certain skein relations. This techinique has been successfully exploited by Bisch and Jones in the classification of subfactors [BJ00] [J:14], and by Scott Morrison and various authors in the construction of subfactors.

In particular, planar algebras have been successfully used to re-construct the Haagerup subfactor [Pet10], and to discover the entirely new extended Haagerup and 4442 subfactors [M:10] [M:17]. However, there are still no known skein theoretic constructions for the Asaeda-Haagerup subfactor and for Izumi's generalized Haagerup subfactors, known as the "$3^G$" subfactors (with $G$ an abelian group).

A skein theoretic approach to these subfactors is important because aside from its inherent mathematical interest, such an approach would allow generalizations to related examples and yield new constructions of subfactors.

In this project we will analyze the graph planar algebras of the exotic subfactors to discover new skein relations. Scott Morrison and Vaughan Jones are leading experts in this technique.

**Problem 6.** *Find planar algebra presentations of the Asaeda-Haagerup and $3^G$ subfactors.*

**Computing the quantum doubles of the exotic subfactors.** The quantum double of a finite depth subfactor $N \subseteq M$ is the fusion category associated to the asymptotic inclusion of the subfactor. It is equivalent to the Drinfeld center of the principal even part of the original subfactor. While this invariant has been studied by numerous researchers in operator algebras [Ocn94, EK95]- as well as by algebraists in the context of abstract tensor categories [Maj91, KT95] - it is extremely difficult to compute explicitly for all but the simplest examples.

A general machinery to compute the quantum double from a structure called a tube algebra was developed by Izumi in [Izu00, Izu01]; he used this machinery to compute the double of the Haagerup subfactor. This approach was successfully applied to the near-group subfactors by Evans and Gannon in [EG12]. To compute the tube algebra one needs certain structure constants for the subfactor called 'quantum 6j symbols'. Computing the quantum double of the Asaeda-Haagerup subfactor was first attempted by Asaeda in her PhD thesis. While she determined some information about the double, she was not able to explicitly compute it.

In this project we will compute the 6j-symbols of the Asaeda-Haagerup subfactor and compute the quantum double. We will then apply this machinery to other exotic subfactors, including the extended Haagerup subfactor and the 4442 subfactor. Pinhas Grossman has worked extensively on determining structure constants in the connection formalism for the Asaeda-Haagerup subfactor. Scott Morrison along with collaborators will work on realising this formalism in the framework of planar algebras, thus facilitating more powerful computer calculations.

**Problem 7.** *Develop a framework for computing 6j symbols from the graph planar algebra of a subfactor.*

**Problem 8.** *Find the quantum doubles of exotic subfactors, including the Asaeda-Haagerup and extended Haagerup subfactors.*

**Classification of families of subfactors.** Combining the results of the previous stages of the project will give us a large stock of new examples of subfactors, as well as a framework for classifying their symmetries. In the final stage of the project we will identify and classify general families of subfactors encompassing the exotic subfactors, using the results of our analyses of the known exotic subfactors along with constructions coming from planar algebras and operator algebras. In particular, we will unify several constructions coming from these different areas in a common algebraic and cominatorial framework and describe invariants to classify subfactor symmetries.

**Problem 9.** *Formulate classes of subfactors that encompass the known exotic subfactors in terms of symmetries of their invariants.*

**Problem 10.** *Construct new exotic subfactors using these symmetries.*

**Timeline** In the first six months of the project we will attempt to construct the missing Asaeda-Haagerup subfactors and find solutions for the Izumi equations for larger Abelian groups. In the next six months we will study generalizations of the Izumi-Haagerup series. In the second year of the project we will study invariants of these exotic subfactors, including the Brauer-Picard groupoids, the quantum doubles, and the skein theory of their planar algebras. In the final year of the project we will study the classification of families of subfactors according to the symmetries of these invariants.

**Project Quality and Innovation** Subfactors are a modern area of mathematical research and their symmetries are still not well understood. The exotic subfactors in particular represent new mathematical structures that have not yet been encountered in other mathematical disciplines. This project will make major progress towards the understanding of subfactors, both by uncovering the symmetries of known subfactors and by constructing new examples of subfactors.

**Feasibility and Benefits** We are well positioned to succeed in this project since our approach follows naturally from our prior work. Scott Morrison has been a major force in both the classification of small index subfactors and in the construction of new exotic subfactors; these projects have already met with outstanding success. Pinhas Grossman has made significant contributions to understanding the structure of subfactor symmetries using the representation theory of tensor categories; this work has led to the discovery of new subfactors related to the known exotic subfactors, as well as uncovering unexpected symmetries in the previously mysterious Asaeda-Haagerup subfactor. Vaughan Jones is the leading world expert on subfactors.

This project will establish Australia as a major international centre for subfactor research. The research has applications in other areas of mathematics, especially the study of von Neumann algebras, fusion and tensor categories, and representation theory. The exotic subfactors which we are attempting to study product certain exotic topological quantum field theories, and are thus interesting to condensed matter physicists studying topological phases of matter, and topological quantum computation.

**Significance and National Research Priority** This project falls within National Research Priority "Frontier technologies and transforming Australian industries: Breakthrough science".

**Collaboration** We have active collaborations both in academia and industry, many of which are focused on the subject matter of this proposal. The research associate for this project will benefit directly from these collaborations, and will be able to quickly establish international connections. A number of these collaborators will visit ANU during the course of this project, building strong ties between the MSI and the international community of researchers in category theory and operator algebras. The proposed project will benefit from and build upon the growing connections between Australian mathematics and international researchers in von Neumann algebras, subfactors and fusion categories. Our existing collaborative relationships with international mathematicians which are most relevant to this project are listed below.

- Prof. Stephen Bigelow (UC Santa Barbara, USA), is an expert in knot theory and representation theory.
- Prof. Dietmar Bisch (Vanderbilt, USA) is an expert in operator algebra, specifically subfactors and their representation theory.
- Prof. Frank Calegari (Northwestern, USA) is an expert in number theory with whom Scott Morrison has collaborated applying unexpected number theoretic techniques to the classification of subfactors and fusion categories.
- Dr. Chris Douglas (Oxford, UK) is an expert in category theory and topological quantum field theory.
- Prof. Masaki Izumi (Kyoto, Japan) is an expert on operator algebras, and in particular a leading figure in the construction of exotic examples of subfactors. We have both collaborated with Prof. Izumi on the classificaton of subfactors.
- Dr. David Jordan (UT Austin, USA) is an expert on the application of algebraic topology to the study of tensor categories. Pinhas Grossman has collaborated with Dr. Jordan on constructing extensions of subfactors and will continue to work with him.
- Dr. Emily Peters (Northwestern, USA) is a postdoctoral researcher studying planar algebras, whose techniques have inspired recent constructions of exotic subfactors.
- Prof. Noah Snyder (Indiana, USA) is an expert on fusion categories, subfactors and number theory. We have both collaborated extensively with Prof. Snyder.
- Dr. Kevin Walker (Microsoft Station Q, USA) is an expert in topological quantum field theory and category theory, working for Microsoft on topological quantum computing.

**Communication of results** The results of the proposed research will be communicated widely through oral presentations at international and national conferences and workshops, and in written form in top-quality journals. The project will involve the coding of custom algorithms for the study of fusion categories and subfactors, and these will be released under an open source license.

## Research environment

The two chief investigators are both young researchers in the early stages of their careers, working at two of Australia's premier universities. Scott Morrison arrived at ANU in 2012, while Pinhas Grossman began his position at UNSW at the beginning of 2013. Subfactor theory, which is our research focus and the subject of this project, is an exciting new area of mathematical research with deep connections to a broad array of diverse mathematical disciplines, as well as to physics. This area has heretofore been sparsely represented in Australia.

This project will establish Australia as a centre for subfactor research. Vaughan Jones is one of the world's pre-eminent mathematics and a leader in research on subfactors. His participation as a partner investigator will bring extensive expertise, and his visits to Australia under this project will contribute to research across the field of operators algebras. We will bring our international collaborators from several continents to visit both ANU and UNSW, forming enduring links between these international researchers and the Australian mathematical community. We intend that this project will be the seed for further developing this international research subject within Australia, by involving students at both undergraduate and postgraduate level, funding a research associate position, and bringing many international visitors.

### Scott Morrison - The Australian National University

Scott Morrison recently began a continuing teaching/research Level C position in the Mathematical Sciences Institute at the ANU, and is particularly excited about the world class research environment here. He chose to come to ANU over competitive offers in Australia and the United States. He received a Discovery Early Career Researcher Award for "Fusion categories and topological quantum field theory" in the 2012 round.

In the 2010 ERA Institution Report, Pure Mathematics and Mathematical Physics at ANU both received a score of 5, the highest possible ranking. In the 2007 Shanghai Jiao Tong rankings of universities by field, ANU was ranked 38th in the world in natural sciences and mathematics (the only Australian university in the top 100 in this category).

Within Mathematical Sciences Institute (MSI) there are 7 research groups. The relevant ones for this project are Algebra & Topology (category theory and quantum topology) and Mathematical Physics (quantum field theory, and applications of category theory to condensed matter physics). These groups represent priority research areas for MSI. The proposed project will benefit from the proximity of a number of researchers at the MSI with relevant expertise, particularly Prof. Amnon Neeman, Prof. Peter Bouwknegt, Prof. Alan Carey, Dr. Bai-Ling Wang, Dr. Anthony Licata, and Dr. Joan Licata.

Dr. Morrison has had extensive discussions with the Director, Thierry Coulhon, and is confident that MSI provides a supportive and productive environment for the research proposed here that is the best possible in Australia. This proposal will support a research associate to participate in the project, helping to build a local research group with critical mass.

Over the next three years, he will continue his research supervision activities, by incorporating PhD students in the project proposed here. His ability to attract PhD students will be aided by the College of Physical Science guaranteeing scholarships to all qualified applicants, and also by ANU's Bachelor of Philosophy (PhB) degree in Science, an elite research-oriented undergraduate degree open to the top 1% of students.

The Mathematical Sciences Institute is engaging in a generational change with coordinated appointment of younger mathematicians representing a broad cross section of modern mathematical research areas. The Mathematical Sciences Institute is therefore actively involved in succession planning in which younger people are needed to take up leadership roles both at ANU and in a national context. At age 33 Morrison is in an age group not well represented in Australian mathematics in general or in the MSI in particular. He has a strong research track record and an international reputation, so the MSI is anticipating moving Scott Morrison into a leadership role at an early stage given the large number of retirements anticipated in the next few years. The award of this Discovery Project to support a research associate would support his preparation for research leadership roles in MSI and nationally.

**Pinhas Grossman - The University of New South Wales**

Pinhas Grossman has just started a continuing teaching/research Level B position at the University of New South Wales. He is a member of the Department of Pure Mathematics in the School of Mathematics and Statistics. UNSW ranked first in Australia in 2012 in the Academic Ranking of World Universities and the School of Mathematics is the largest such centre in Australia. The School has researchers working in pure mathematics, applied mathematics, and statistics; it also has many links with industry.

The Department of Pure Mathematics at UNSW is particularly strong in the area of modern analysis, an exciting branch of mathematical research that includes operator algebras, one of the most important tools we will be using in our project. Working in the department are a number of notable experts in operator theory and operator algebras. Relevant faculty for the subject matter of this project include Prof. Ian Doust, Prof. Fedor Sukochev, Prof. Norman Wildberger, and Dr. Denis Potapov.

UNSW is located in Sydney, which also houses several other major Australian universities. Among these is the University of Sydney, where many outstanding mathematicians work in pure mathematics, including areas relevant to this project such as representation theory and Hecke algebras. Dr. Grossman will be organising the joint UNSW/University of Sydney Colloquium this year, which will be an excellent opportunity to develop collaborations with mathematicians working at the University of Sydney.

**Vaughan F. R. Jones - Vanderbilt University**

Vaughan Jones is a Distinguished Professor at the department of mathematics at Vanderbilt University. He is a member of the Vanderbilt's Center for Noncommutative Geometry and Operator Algebras. As a winner of the Fields Medal in 1990, mathematics' highest honour, he is one of the world's pre-eminent mathematicians.

The MSI at ANU plans to establish a stronger relationship with Vanderbilt University where, in addition to Jones, Fields medalist Alain Connes and several other prominent operator algebraists hold positions. This project will be a significant step towards building that relationship.

## Role of personnel

As Project Leader Scott Morrison will be responsible for the overall management and supervision of the project. He has expertise in all the core areas of the project, including tensor categories, subfactors, planar algebras and higher categories.

As Chief Investigator Pinhas Grossman will be involved in all aspects of the project, and responsible for the successful completion of project goals. His expertise on subfactors and the representation theory of fusion categories will be critical to successful completion.

As Partner Investigator, Vaughan Jones will be essential in tackling the goals of the project, and in particular his expertise and track record analyzing the algebraic structure of subfactors and planar algebras will be critical to understanding the skein theory of exotic subfactors, and finding new operator algebraic constructions of group-like subfactors and generalizations thereof.

The research assistant will also have expertise in one or more of these areas, and well as providing additional expertise in number theory and/or quantum groups and/or operator algebra. Ideally the research assistant will have experience with mathematical computing and symbolic computer algebra packages. As the project requires studying many examples it will be important to develop computational tools to study them systematically. They will be trained in additional areas as required and will work on the project in close collaboration with and with appropriate direction from the chief investigators.

For specific additional expertise, we will be consulting with an international network of collaborators. Those who will be working with us on specific topics are listed above.A large proportion of the proposed research is likely to end up jointly authored with at least one of the collaborators listed above, and/or the RA, and/or PhD students.

**International Collaboration Award** The partner investigator, Prof. Vaughan Jones, is located overseas. The chief investigators will work with him in person during his visits to Australia and at other times will communicate electronically. While many aspects of mathematical research can be easily be conducted purely via electronic communications, the creative process of developing new ideas and conjectures, and exploring the possibilities for proofs, is strongly supported by the possibility of additional in-person collaborative time. We therefore request ICA's for each of the chief investigators to spend three months in the USA over the course of the project in intensive collaboration with Prof. Jones.

## References


[AH99] Marta Asaeda and Uffe Haagerup, *Exotic subfactors of finite depth with Jones indices* $(5+\sqrt{13})/2$ *and* $(5+\sqrt{17})/2$, Comm. Math. Phys. **202** (1999), no. 1, 1–63, MR1686551 arXiv:math.OA/9803044.

[BJ97] Dietmar Bisch and Vaughan F. R. Jones, *Algebras associated to intermediate subfactors*, Invent. Math. **128** (1997), no. 1, 89–157, MR1437496.

[BJ00] Dietmar Bisch and Vaughan F. R. Jones, *Singly generated planar algebras of small dimension*, Duke Math. J. **101** (2000), no. 1, 41–75, MR1733737.

[BN91] Jocelyne Bion-Nadal, *An example of a subfactor of the hyperfinite* $\mathrm{II}_1$ *factor whose principal graph invariant is the Coxeter graph* $E_6$, Current topics in operator algebras (Nara, 1990), World Sci. Publ., River Edge, NJ, 1991, MR1193933, pp. 104–113.

[EG11] David E. Evans and Terry Gannon, *The exoticness and realisability of twisted Haagerup-Izumi modular data*, Comm. Math. Phys. **307** (2011), no. 2, 463–512, arXiv:1006.1326 MR2837122.

[EG12] David E. Evans and Terry Gannon, *Near-group fusion categories and their doubles*, 2012, arXiv:1208.1500.

[EK95] David E. Evans and Yasuyuki Kawahigashi, *On Ocneanu's theory of asymptotic inclusions for subfactors, topological quantum field theories and quantum doubles*, Internat. J. Math. **6** (1995), no. 2, 205–228, MR1316301.

[ENO09] Pavel Etingof, Dmitri Nikshych, and Victor Ostrik, *Fusion categories and homotopy theory*, 2009, With an appendix by Ehud Meir. arXiv:0909.3140.

[GdlHJ89] Frederick M. Goodman, Pierre de la Harpe, and Vaughan F. R. Jones, *Coxeter graphs and towers of algebras*, Mathematical Sciences Research Institute Publications, vol. 14, Springer-Verlag, New York, 1989, MR999799.

[GJS12] Pinhas Grossman, David Jordan, and Noah Snyder, *Cyclic extensions of fusion categories via the brauer-picard groupoid*, 2012, arXiv:1211.6414.

[GS12] Pinhas Grossman and Noah Snyder, *The Brauer-Picard group of the Asaeda-Haagerup fusion categories*, 2012, arXiv:1202.4396.

[Haa94] Uffe Haagerup, *Principal graphs of subfactors in the index range $4 < [M : N] < 3 + \sqrt{2}$*, Subfactors (Kyuzeso, 1993), World Sci. Publ., River Edge, NJ, 1994, MR1317352, pp. 1–38.

[Han10] Richard Han, *A construction of the "2221" planar algebra*, Ph.D. thesis, University of California, Riverside, 2010, arXiv:1102.2052.

[Izu91] Masaki Izumi, *Application of fusion rules to classification of subfactors*, Publ. Res. Inst. Math. Sci. **27** (1991), no. 6, 953–994, MR1145672.

[Izu93] ———, *Subalgebras of infinite $C^*$-algebras with finite Watatani indices. I. Cuntz algebras*, Comm. Math. Phys. **155** (1993), no. 1, 157–182, MR1228532.

[Izu94] ———, *On flatness of the Coxeter graph $E_8$*, Pacific J. Math. **166** (1994), no. 2, 305–327, MR1313457 euclid.pjm/1102621140.

[Izu98] ———, *Subalgebras of infinite $C^*$-algebras with finite Watatani indices. II. Cuntz-Krieger algebras*, Duke Math. J. **91** (1998), no. 3, 409–461, MR1604162.

[Izu00] ———, *The structure of sectors associated with Longo-Rehren inclusions. I. General theory*, Comm. Math. Phys. **213** (2000), no. 1, 127–179, MR1782145.

[Izu01] ———, *The structure of sectors associated with Longo-Rehren inclusions. II. Examples*, Rev. Math. Phys. **13** (2001), no. 5, 603–674, MR1832764.

[Jon83] Vaughan F. R. Jones, *Index for subfactors*, Invent. Math. **72** (1983), no. 1, 1–25, MR696688.

[Jon85] ———, *A polynomial invariant for knots via von Neumann algebras*, Bull. Amer. Math. Soc. (N.S.) **12** (1985), no. 1, 103–111, MR766964.

[Jon87] ———, *Hecke algebra representations of braid groups and link polynomials*, Ann. of Math. (2) **126** (1987), no. 2, 335–388, MR0908150.

[Jon99] ———, *Planar algebras, I*, 1999, arXiv:math.QA/9909027.

[JS91] André Joyal and Ross Street, *Tortile Yang-Baxter operators in tensor categories*, J. Pure Appl. Algebra **71** (1991), no. 1, 43–51, MR1107651.

[JT84] Vaughan F. R. Jones and Masamichi Takesaki, *Actions of compact abelian groups on semifinite injective factors*, Acta Math. **153** (1984), no. 3-4, 213–258, MR0766264.

[Kaw95] Yasuyuki Kawahigashi, *On flatness of Ocneanu's connections on the Dynkin diagrams and classification of subfactors*, J. Funct. Anal. **127** (1995), no. 1, 63–107, MR1308617.

[KO02] Alexander Kirillov, Jr. and Viktor Ostrik, *On a $q$-analogue of the McKay correspondence and the ADE classification of $\mathfrak{sl}_2$ conformal field theories*, Adv. Math. **171** (2002), no. 2, 183–227, MR1936496 arXiv:math.QA/0101219.

[KT95] Christian Kassel and Vladimir Turaev, *Double construction for monoidal categories*, Acta Math. **175** (1995), no. 1, 1–48, MR1353016.

[Maj91] Shahn Majid, *Representations, duals and quantum doubles of monoidal categories*, Proceedings of the Winter School on Geometry and Physics (Srní, 1990), no. 26, 1991, MR1151906, pp. 197–206.

[Ocn88] Adrian Ocneanu, *Quantized groups, string algebras and Galois theory for algebras*, Operator algebras and applications, Vol. 2, London Math. Soc. Lecture Note Ser., vol. 136, Cambridge Univ. Press, Cambridge, 1988, MR996454, pp. 119–172.

[Ocn94] ———, *Chirality for operator algebras*, Subfactors (Kyuzeso, 1993), World Sci. Publ., River Edge, NJ, 1994, MR1317353, pp. 39–63.

[Pet10] Emily Peters, *A planar algebra construction of the Haagerup subfactor*, Internat. J. Math. **21** (2010), no. 8, 987–1045, arXiv:0902.1294 MR2679382.

[Pop94] Sorin Popa, *Classification of amenable subfactors of type II*, Acta Math. **172** (1994), no. 2, 163–255, MR1278111.

[Pop95] ———, *An axiomatization of the lattice of higher relative commutants of a subfactor*, Invent. Math. **120** (1995), no. 3, 427–445, MR1334479.

[PT12] David Penneys and James Tener, *Classification of subfactors of index less than 5, part 4: vines*, International Journal of Mathematics **23** (2012), no. 3, 1250017 (18 pages), arXiv:1010.3797.

**C2. Strategic Statement by the Administering Organisation**

(For all Proposals requesting a DORA, please provide a Strategic Statement of two A4 pages maximum which outlines the:

(a) extent to which the proposed Project and the DORA Candidate aligns with the existing and/or emerging research strengths of the Administering Organisation; and

(b) arrangements under which the proposed Project, the DORA Candidate, and other investigators will be hosted in a collaborative research environment.)

**Strategic Statement**

| |
|---|
| PDF attachment not submitted |

**D1. What is the proposed budget for your project?**

(Please provide details of the budget proposed for your project.)

**Proposal Funding Summary**

Total requested budget: $406099

Year 1

| Description | ARC | AdminOrg | Other Eligible Organisation | Other Organisation |
|---|---|---|---|---|
| Direct Cost | 132533 | 29270 | 53800 | 38000 |
| Personnel | 109433 | 29270 | 53800 | 30000 |
| Pinhas Grossman 40% | 0 | 0 | 53800 | 0 |
| Research associate (Level B2) | 107229 | 2773 | 0 | 0 |
| Scott Morrison 40% | 0 | 26425 | 0 | 0 |
| Mathematical computing support | 2204 | 72 | 0 | 0 |
| Vaughan Jones 10% | 0 | 0 | 0 | 30000 |
| Equipment | 4600 | 0 | 0 | 0 |
| specialized computer for research associate | 4000 | 0 | 0 | 0 |
| computer algebra system licensing | 600 | 0 | 0 | 0 |
| Travel | 18500 | 0 | 0 | 8000 |
| Scott Morrison | 5000 | 0 | 0 | 0 |
| Pinhas Grossman | 6500 | 0 | 0 | 0 |
| Research associate | 2000 | 0 | 0 | 0 |
| Vaughan Jones | 0 | 0 | 0 | 8000 |
| Collaborator's visits - Noah Snyder | 5000 | 0 | 0 | 0 |

Year 2

| Description | ARC | AdminOrg | Other Eligible Organisation | Other Organisation |
|---|---|---|---|---|
| Direct Cost | 139033 | 46270 | 53800 | 38000 |
| Personnel | 109433 | 46270 | 53800 | 30000 |
| Pinhas Grossman 40% | 0 | 0 | 53800 | 0 |
| Research associate (Level B2) | 107229 | 2773 | 0 | 0 |
| Scott Morrison 40% | 0 | 43425 | 0 | 0 |
| Mathematical computing support | 2204 | 72 | 0 | 0 |
| Vaughan Jones 10% | 0 | 0 | 0 | 30000 |
| Equipment | 1600 | 0 | 0 | 0 |
| specialized computer for research associate | 1000 | 0 | 0 | 0 |
| computer algebra system licensing | 600 | 0 | 0 | 0 |
| Travel | 18000 | 0 | 0 | 8000 |
| Scott Morrison | 5000 | 0 | 0 | 0 |
| Pinhas Grossman | 6500 | 0 | 0 | 0 |
| Research associate | 2000 | 0 | 0 | 0 |
| Vaughan Jones | 0 | 0 | 0 | 8000 |

| Description | ARC | AdminOrg | Other Eligible Organisation | Other Organisation |
|---|---|---|---|---|
| Collaborator's visit - Masaki Izumi | 4500 | 0 | 0 | 0 |
| International Collaboration Award | 10000 | 0 | 0 | 0 |
| Scott Morrison | 5000 | 0 | 0 | 0 |
| Pinhas Grossman | 5000 | 0 | 0 | 0 |

Year 3

| Description | ARC | AdminOrg | Other Eligible Organisation | Other Organisation |
|---|---|---|---|---|
| Direct Cost | 134533 | 66325 | 53800 | 38000 |
| Personnel | 109433 | 66325 | 53800 | 30000 |
| Pinhas Grossman 40% | 0 | 0 | 53800 | 0 |
| Research associate (Level B2) | 107229 | 2773 | 0 | 0 |
| Scott Morrison 40% | 0 | 63480 | 0 | 0 |
| Mathematical computing support | 2204 | 72 | 0 | 0 |
| Vaughan Jones 10% | 0 | 0 | 0 | 30000 |
| Equipment | 1600 | 0 | 0 | 0 |
| specialized computer for research associate | 1000 | 0 | 0 | 0 |
| computer algebra system licensing | 600 | 0 | 0 | 0 |
| Travel | 13500 | 0 | 0 | 8000 |
| Scott Morrison | 5000 | 0 | 0 | 0 |
| Pinhas Grossman | 6500 | 0 | 0 | 0 |
| Research associate | 2000 | 0 | 0 | 0 |
| Vaughan Jones | 0 | 0 | 0 | 8000 |
| International Collaboration Award | 10000 | 0 | 0 | 0 |
| Scott Morrison | 5000 | 0 | 0 | 0 |
| Pinhas Grossman | 5000 | 0 | 0 | 0 |

**Other Eligible Organisation Summary**

| | Year 1 | Year 2 | Year 3 |
|---|---|---|---|
| The University of New South Wales | 53800 | 53800 | 53800 |
| **Total** | **53800** | **53800** | **53800** |

**Other Organisation Summary**

| | Year 1 | Year 2 | Year 3 |
|---|---|---|---|
| Vanderbilt University | 38000 | 38000 | 38000 |
| **Total** | **38000** | **38000** | **38000** |

## PART E - Budget Justifications (DP140100732)

### E1. Justification of funding requested from the ARC (excluding justification of Discovery Outstanding Researcher Award requests)

(In no more than three A4 pages fully justify in terms of need and cost, each budget item requested from the ARC (use the same headings as in the ARC Request Budget Column). NOTE: Justification for Discovery Outstanding Researcher Award requests should be made in Part F – Personnel.)

# E1 Justification of funding requested from the ARC

## Personnel

The principal expense of this project will be three years of salary for a research associate to support the project. The research associate will be a mathematician with up to 3 years postdoctoral experience. The research associate will work on all aspects of the proposed project, working closely with both chief investigators. We intend that they will be based at the ANU, although given the proximity to Sydney we anticipate that regular collaborative meetings with Pinhas Grossman at UNSW will be an essential part of the project. The research associate salary component of this project offers excellent value for money. It will be providing a postdoctoral research position in an international active field, with mentorship provided by leading researchers. Although subfactors are not a research field well represented in Australia at this point, the recent arrival of the two chief investigators in the country offers the opportunity to establish a very active research node. The involvement of Fields Medallist Vaughan Jones as partner investigator will support strong international connections. The provision of support for a postdoctoral research associate will cement this development, supporting an early career researcher and integrating them into ongoing research program. The research associate will have expertise in category theory and operator algebra, together with at least one of number theory and representation theory. Facility with mathematical computation will also be valuable. We have budgeted for a level B2 salary, plus 28% on-costs, with a 3.31% contribution from ANU to reach the 31.31% on cost rate.

We are also budgeting for mathematical computing support at 2 hours per fortnight, at the appropriate ANU rate of $32.70. With on costs, this comes to $2,204 for each year. This is required for the development of new computer algorithms required for the computational aspects of the project, and database management. The MSI will contribute the additional required 3.31% on cost.

## Equipment

The execution of this project will require specialised computing equipment and software. As such, we will need to provide a computer with the capacity to run advanced computer algebra packages to the research associate, as well a long-running hand-coded calculations. This will require fast processors and more internal memory than a standard desktop computer. Parts of the project will require intensive computation. We are budgeting $4,000 in the first year to purchase a customized computer for the research associate, and $1,000 in each subsequent year for hardware upgrades. We will also need to purchase licenses for certain specialised mathematical software, for the investigators, research associate, and students, which we have budgeted at $600 in each year. This budget for specialised computer equipment will help ensure the successful completion of the difficult combinatorial searches required for the project.

## Travel - International

We are requesting support for travel costs for both Chief Investigators and for the research associate. This project involves collaborations with a significant number of international mathematicians, the United States, Japan and the United Kingdom (see §C: Collaboration). We will each be traveling internationally to work with these collaborators and to attend conferences to present the results of this work and to network with new collaborators. The research associate will also need to be present for some of these meetings, and will need to travel to conferences to disseminate the results of the project and to develop their own international standing in the field.

The research field is internationally active, and many of the relevant conferences are in the United States, Japan and France. We have been regularly invited to present research at the highest calibre conferences, and anticipate that our research associate will also participate at these conferences.

**Pinhas Grossman** will be traveling as part of this project to meet with international collaborators and to attend research conferences, as well as to meet with the other investigators on the project. In 2014, he will travel to the US for 2 weeks to meet for one week each with collaborators Noah Snyder and David Jordan, who are located at Indiana University and the University of Texas at Austin, respectively. He will also attend a Banff workshop on subfactors and fusion categories which is being organised by Scott Morrison.

Indicative expenses for this travel is as follows. For the visit to the United States, indicative airfare in December (the likely month for that visit) to Indianapolis is $2001 on United Airlines. We estimate per diem and accommodation expenses as $1,200–$1,800 per week (using the relevant ATO ruling for reasonable travel expenses, and $150 per night accommodation) to arrive at a cost of $5000 for the trip. For the trip to Banff, we use the indicative airfare of $1406 on United Airlines to Calgary in March plus $110 for the shuttle to Banff. Accommodation and meals will be provided by the conference, for a total trip cost of $1516.

This is a total of $6517 for Pinhas Grossman's travel in 2014, and similar travel expenses are anticipated for each of the three years of the project. Therefore we budget $6500 per year for Pinhas Grossman's travel.

**Scott Morrison** will also be traveling for this project, to meet with international collaborators and to attend research conferences. In 2014 Scott Morrison will travel to Banff for the workshop on subfactors he is organising there; as for Pinhas Grossman, we estimate the cost of this travel at $1516. He will also make a trip to the US for one week to consult with Dr. Emily Peters at Northwestern University. For the visit to the United States, indicative airfare to to Chicago in January 2014 is $2046 on Air Pacific Limited. We estimate per diem and accommodation expenses as $1,200–$1,800 per week (using the relevant ATO ruling for reasonable travel expenses, and $150 per night accommodation) to arrive at a cost of $3546 for the trip.

Scott Morrison's total travel expenses in 2014 will thus be $5062, and we anticipate similar travel expenses in 2015 and 2016. Therefore we budget $5000 per year for Scott Morrison's travel expenses.

**The research associate** will also attend one international conference per year. In 2014 he will attend the Banff workshop for a cost of $1516 as above. In 2015 and 2016 he will attend one international conference each. These are likely to be slightly more expensive since unlike in Banff, accommodation is generally not provided by the conference. We therefore budget a total of $6000 over the life of the project for the research associate's travel.

**Collaborators** We also plan to invite our collaborators Professor Masaki Izumi of Kyoto University, Japan and Professor Noah Snyder of Indiana University, USA to work on the project with us in Australia. Prof. Snyder will visit during Year 1 of the project, and Prof. Izumi will visit during Year 2 of the project. They will each spend one week at ANU and one week at UNSW. Indicative expenses are as follows. For the flight from Osaka to Sydney we estimate $1250 (c.f. recent quotes for January of $1245 on Korean Air and $1264 on Malaysia Airlines). For the flight from Indianapolis to Sydney we estimate $2000 (cf. recent quotes for December of $2035 on Qantas Airways and $2090 on Delta). For each week of the visits, we estimate the cost of per diem expenses and accommodation as $1,200–$1,800 per week (using the relevant ATO ruling for reasonable travel expenses, and $150 per night accommodation). Since there are two visitors each coming for two weeks, we estimate the total cost of these visits at $9500.

## International Collaboration Award

We are requesting International Collaboration Awards for each of the Chief Investigators to spend a total of three months at Vanderbilt University in the US over the course of the project for intensive collaboration with Professor Jones. Mathematical research requires a combination of face-to-face time spent with collaborators for discussions of new ideas, and time spent alone for introspection and calculation.

Collaboration can also be achieved through electronic communication, but at this point nothing can really replace being in the same room, bouncing new ideas and arguing through the intricacies of proofs. Although the project is certainly feasible if in-person collaboration with Prof. Jones is limited to his visits to Australia and meetings at conferences, we feel that the project will be significantly supported by International Collaboration Awards providing for additional travel from Australia to Vanderbilt, and that this extra support would offer excellent value for money. Weekly expenses are estimated at \$1200-\$1800 as above, and airfare from Sydney to Nashville is upwards of \$2000 (cf. recent quote of \$2013 in December on United). We therefore request the maximum award pro-rated to \$10000 for three months, for each of the two chief investigators.

## E2. Details of non-ARC contributions

(In no more than one A4 page provide an explanation of how non-ARC contributions will support the proposed project (use the same headings as in the non-ARC contributions Budget Column).)

**Attached PDF**

# E2 Details of non-ARC contributions

## Personnel

The Mathematical Sciences Institute will provide the additional $2,773 per year to cover the gap between ARC on costs of 28% and ANU on costs of 31.31% on the research associate salary. The MSI also provides stipends for qualified Ph.D. students to work with Scott Morrison on the project.

The Mathematical Sciences Institute is covering the gap between Scott Morrison's DECRA salary and his ANU salary at Level C4 with loading and 31.31% on costs. The level C4 salary with loading is $115,044, and $151,064 with on costs. We are calculating 2016 salaries on the assumption of an increment to Level C6, bringing the total with on costs to $158,701. Because the DECRA lasts for the first 18 months of this proposed project, we are accounting this as a contribution of $85,000 in 2014 and $42,500 in 2015. Thus the gap gap is $66,064 in 2014, $108,564 in 2015 and $158,701 in 2016. I have recorded these amounts at 40% FTE in the budget table. The MSI will cover the additional 3.31% on cost gap for mathematical computing support, for $72 per annum.

The School of Mathematics and Statistics will contribute Pinhas Grossman's UNSW salary at Level B4 at 40% FTE. This salary will be $105,079 from January 2014, excluding superannuation. Thus the University of New South Wales is contributing $53,800 ( = $105,079 x 0.40 x 1.28, to include on-costs) in each of the three years.

Vanderbilt University's salary contribution for Vaughan Jones while he works on this project is $30,000 in each year of the project.

## Travel

Vaughan Jones, as an international partner investigator, will be traveling to Sydney and Canberra to meet with the chief investigators. As his position as Distinguished Professor at Vanderbilt University includes generous travel arrangements, we have not requested any travel money from the ARC. Vanderbilt's contribution is $8,000 each year, for a two week visit to Australia.

## F1. Personal details

(The personal details will be filled out for you automatically. To update any of your personal details in this form, please update your profile accordingly and your details will update automatically in this form.)

**Title**

Doctor

**Family Name**

Morrison

**First Name**

Scott

**Person identifier**

69391562

**Role**

Chief Investigator

## F2. Postal address

(The postal address will be filled out for you automatically. To update your postal address, please update your profile accordingly and your postal address will update automatically in this form.)

**Postal Address Line 1**

22/7 Ijong St

**Locality**

Braddon

**State**

ACT

**Postcode**

2612

**Country**

Australia

**F3. Are you a current member of the ARC or its selection or other advisory committees?**

(This relates only to ARC College or Selection Advisory Committee members for National Competitive Grants Program funding schemes.)

**Current Member of Advisory Committee**

| |
|---|
| No |

**F4. Please name any of your relatives or close social/professional associates that are members of the ARC or its selection or other advisory committees.**

| | Title | First Name | Family Name |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

**F5. Please name any Commonwealth-funded Research Centres that you will be associated with as at 1 January 2014.**

| | Full Legal Name of Centre | Start Date | Cessation Date | Centre Role |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |

| | Centre Role if Other |
|---|---|
| 1 | |
| 2 | |

**F6. Are you an Indigenous Participant?**

**Indigenous Participant**

| |
|---|
| No |

**F7. Awarded ARC Fellowships and ARC Awards**

(Please provide the name and the funding period for any awarded ARC Fellowship(s) or ARC Award(s).
(For example: ARF 2005-2009, DORA 2012-2014))

| |
|---|
| DECRA 2012-2015 |

**F8. PhD Qualification**

**F8.1. Do you hold a PhD or expect to be awarded a PhD qualification in the near future?**

**PhD Yes/No**

| |
|---|
| Yes |

**F8.2. If you hold a PhD or expect to be awarded a PhD qualification in the near future, please enter the date your PhD has been awarded or the date your thesis will be submitted, respectively.**

**Date of Award**

01/07/2007

**F9. Qualifications**

| | Degree/Award | Year | Discipline/Field | Organisation Name |
|---|---|---|---|---|
| 1 | PhD | 2007 | Mathematics | University of California, Berkeley |
| 2 | BSc (Hons) | 2001 | Mathematics | The University of New South Wales |

| | Country |
|---|---|
| 1 | United States |
| 2 | Australia |

**F10. Current and previous appointment(s)/position(s) – during the past 10 years**

| | Position | Organisation Name | Department | Year Appointed |
|---|---|---|---|---|
| 1 | ARC Fellow / Senior Lecturer | The Australian National University | Mathematical Sciences Institute | 2012 |
| 2 | Miller Fellow | University of California, Berkeley | Miller Institute for Basic Research and the Department of Mathematics | 2009 |
| 3 | Postdoctoral Researcher | Microsoft Pty Ltd | Microsoft Station Q | 2007 |

| | Continuity | Employment Kind | Current |
|---|---|---|---|
| 1 | Permanent | Full Time | Yes |
| 2 | Contract | Full Time | No |
| 3 | Contract | Full Time | No |

**F11. Organisational affiliations for eligibility purposes for this Proposal**

(Name of the organisation you will be associated with for the purposes of satisfying the eligibility requirements for your nominated role in undertaking the proposed research (i.e. for a CI this will usually be the Eligible Organisation at which they will employed or hold an adjunct appointment as at 1 January 2014 and beyond; for PIs it will generally be their main employer as at 1 January 2014).)

**Organisation Name**

The Australian National University

**Type of Affiliation**

Employee

**F12. What is your time commitment (%FTE) to this Project?**

40

**F13. Are you requesting an International Collaboration Award?**

(Note: If you are an Australian-based PI, you must choose 'No'. Also, if you are a PI working in an Australian Eligible Organisation overseas campus you must choose 'No'.)

**International Collaboration Award**

| |
|---|
| Yes |

**F14. Discovery Outstanding Researcher Award**

**F14.1. Justification**

(In no more than two A4 pages please justify how this Discovery Outstanding Researcher Award would benefit, enhance and expedite the overall Project.)

**Justification**

| |
|---|
| PDF attachment not submitted |

**F15. Research Record Relative to Opportunities**

**F15.1. Details on your career and opportunities for research over the last 10 years.**

(Write a maximum of 5250 characters (approx. 750 words).

Provide and explain:
(i) The number of years it has been since you graduated with your highest educational qualification;
(ii) The research opportunities that you have had in the context of your employment situation, the research component of your employment conditions, and any unemployment or part-time employment you may have had;
(iii) Whether you are a research-only, teaching and research, teaching only, teaching and administration, research and administration, or administration–only academic, as well as giving any additional information (for example, part-time status) needed to understand your situation. Give an indication of what percentage of time you have spent over the last ten years in those roles;
(iv) Any career interruptions you have had for childbirth, carer’s responsibility, misadventure, or debilitating illness;
(v) The research mentoring and research facilities available to you; and
(vi) Any other aspects of your career or opportunities for research that are relevant to assessment and that have not been detailed elsewhere in this Proposal (for example, any circumstances that may have affected the time you have had to conduct and publish research).)

I received my Ph.D. 6 years ago, in May 2007, and since then I have enjoyed unparalleled research opportunities in academia and industry.

* UC Berkeley, Ph.D. May 2007.
During my Ph.D. I worked with Fields Medallist Vaughan Jones. I received the Herb Alexander Prize for Best Thesis in Pure Mathematics, for my thesis on "A Diagrammatic Category for the Representation Theory of U_q(sl_n)".

* Microsoft Station Q, April 2007--June 2009.
I accelerated the completion of my Ph.D. work because Microsoft offered me a postdoctoral research-only position at 'Station Q', situated at UC Santa Barbara. This research groups spans pure mathematics, mathematical physics and condensed matter physics, and works on projects focused on the development of topological quantum computing based on physical substrates supporting topological phases of matter. It is directed by Fields Medallist Michael Freedman. I established significant collaborations with mathematicians working in industry. I work on an ongoing basis with members of Station Q, and anticipate further future development of applications for my work and inspiration for relevant new research problems.

* Miller Institute for Basic Research, July 2009--June 2012.
I then became a Miller Fellow at UC Berkeley. This prestigious 3 year research-only postdoctoral fellowship

supported my ongoing research, and allowed me to focus on developing innovative new ideas. This fellowship allowed me to travel to conferences, and develop a strong network of disciplinary and interdisciplinary collaborations

* Australian National University, July 2012---.
I have a continuing teaching/research position as Senior Lecturer in the Mathematical Sciences Institute of the Australian National University. I have been awarded a Discovery Early Career Researcher Award, for 2012-2014, and as such currently have a research only position.

I have been fortunate to have had no interruptions to my career path. I have written 18 papers in the 6 years since my Ph.D., many of which appear in journals with an ERA ranking of A*. (See section F15.2 for more details.) As a result of my work in quantum topology, fusion categories and subfactors I have been invited to speak at many international conferences (e.g. in Sweden, India, France and Japan), and have been a colloquium speaker at Caltech, the University of Oregon, the University of Toronto, Vanderbilt University and at UC Berkeley. I gave a plenary talk at the Australian Mathematical Society annual meeting last year. My research to date has involved a wide network of international collaborators in both academia and industry, including two Fields Medallists.

I receive excellent mentoring from mathematicians at the MSI, including Prof. Amnon Neeman, Prof. Alan Carey, Prof. Peter Bouwknegt, Prof. Andrew Hassell, and Dr. James Borger. As described under "Research Environment" in Section C, the MSI anticipates my being an integral part of the generational change within the MSI and Australian mathematics, taking up leadership roles at ANU and nationally.

**F15.2. Recent significant publications (since 2003)**

(Please attach a PDF with a list of your recent significant publications (40 pages maximum). (1) Provide your research publications published in the last ten years split into the five categories of (a) scholarly books, (b) scholarly book chapters, (c) refereed journal articles, (d) refereed conference papers only when the paper was published in full in the proceedings and, (e) other (for example, major exhibitions, compositions or performances). You must number your publications continuously. Asterisk the publications relevant to this Proposal. (2) Provide a list of your ARC grants awarded in the last ten years on which you have been a Chief Investigator. Give the ARC grant number, Chief Investigator names in the order that they appear on the grant, the amount funded, the years for which the grant was awarded, and the title of the grant. Please refer to the Instructions to Applicants for format requirements. With respect to your numbered publications in the last ten years given in part 1 of question F15.2, next to each ARC grant, provide the numbers of the publications from part 1 of question F15.2 that arose from, or were in part supported by, your ARC grants.)

## F15.2. Recent significant publications (Scott Morrison)

### (1)

I regularly publish in the highest quality journals, including the *Proceedings of the National Academy of Sciences*, *Acta Mathematica*, *Geometry & Topology* and *Communications in Mathematical Physics*. Of my thirteen peer-reviewed papers in ERA ranked journals, eight have been in A*-ranked journals, and a further three in A-ranked journals. I have another two peer-reviewed papers in the excellent new journal of Quantum Topology. Many of my papers have been on questions preliminary to the proposed research, and indicate the considerable existing momentum behind the project.

### Refereed journal articles

*Articles with direct relevance to the proposed project are marked with a *. At the end of each line, the 2010 ERA ranking and 2011 Mathematical Citation Quotient are shown in square brackets. The overall MCQ for 2011 is 0.32; notice that every one of my publications is in a journal with a higher MCQ, and many are significantly higher.*


* **[M:1]** **Scott Morrison** and David Penneys, *Constructing spoke subfactors using the jellyfish algorithm* to appear in Transactions of the American Mathematical Society (accepted 19 Feb 2013), available at arXiv:1208.3637. [A*, 1.05]

* **[M:2]** Masaki Izumi, Vaughan F. R. Jones, **Scott Morrison**, and Noah Snyder, *Subfactors of index less than 5, part 3: quadruple points*. Communications in Mathematical Physics, vol. 316, issue 2 (2012), pp. 531–554, available at arXiv:1109.3190. [A*, 1.38]

* **[M:3]** **Scott Morrison**, David Penneys, Emily Peters and Noah Snyder, *Subfactors of index less than 5, part 2: triple points*. International Journal of Mathematics, vol. 23, No. 3 (2012) 1250016, available at arXiv:1007.2240. [A, 0.51]

* **[M:4]** **Scott Morrison** and Noah Snyder, *Subfactors of index less than 5, part 1: the principal graph odometer*. Communications in Mathematical Physics, vol. 312, issue 1 (2012), pp. 1–35, available at arXiv:1007.1730. [A*, 1.38]

**[M:5]** **Scott Morrison** and Kevin Walker, *Higher categories, colimits and the blob complex*, Proceedings of the National Academy of Sciences, May 17, 2011 vol. 108 no. 20 pp. 8139–8145, available at. [A*, 1.12]

**[M:6]** **Scott Morrison** and Kevin Walker, *The blob complex*. Geometry & Topology 16 (2012) 1481–1607, available at arXiv:1009.5025. [A*, 1.45]

* **[M:7]** Frank Calegari, **Scott Morrison** and Noah Snyder, *Cyclotomic integers, fusion categories, and subfactors*, Communications in Mathematical Physics Vol. 303, Issue 3 (2011), pp. 845–896. Available at arXiv:1004.0665. [A*, 1.38]

* **[M:8]** **Scott Morrison**, Emily Peters and Noah Snyder, *Knot polynomial identities and quantum group coincidences*, Quantum Topology Vol. 2 (2011) pp. 101–156. Available at arXiv:1003.0022. [new journal - unranked, 1.54]

* **[M:9]** **Scott Morrison** and Noah Snyder, *Non-cyclotomic fusion categories*, to appear in Transactions of the American Mathematical Society (accepted 3 Nov 2010), available at arXiv:1002.0168. [A*, 1.05]

* **[M:10]** Stephen Bigelow, **Scott Morrison**, Emily Peters and Noah Snyder, *Constructing the extended Haagerup planar algebra*, Acta Mathematica, vol. 209 (2012), pp. 29–82, available at arXiv:0909.4099. [A*, 3.53]

[**M:11**] **Scott Morrison**, *The braid group surjects onto $G_2$ tensor space*, Pacific Journal of Mathematics, Vol. 249 (2011), No. 1, 189–198. Available at arXiv:0907.0256. [A, 0.58]

[**M:12**] Michael Freedman, Robert Gompf, **Scott Morrison** and Kevin Walker, *Man and machine thinking about the smooth 4-d Poincaré conjecture*, Quantum Topology, Vol. 1, Issue 2 (2010), pp. 171–208. Available at arXiv:0906.5177. [new journal - unranked, 1.54]

* [**M:13**] **Scott Morrison**, Emily Peters and Noah Snyder, *Skein theory for the $\mathcal{D}_{2n}$ planar algebras*, Journal of Pure and Applied Algebra, Vol. 214, No. 2 (2010) pp. 117-139. Available at arXiv:0808.0764. [A, 0.62]

[**M:14**] David Clark, **Scott Morrison** and Kevin Walker, *Fixing the functoriality of Khovanov homology*, Geometry & Topology, 13 (2009) pp. 1499–1582. Available at arXiv:math.GT/0701339. [A*, 1.45]

[**M:15**] **Scott Morrison** and Ari Nieh,*On Khovanov's cobordism theory for $\mathfrak{su}_3$ knot homology*, Journal of Knot Theory and its Ramifications, Vol. 17, No. 9 (2008), pp. 1121–1173. Available at arXiv:math.GT/0612754. [B, 0.38]

[**M:16**] Dror Bar-Natan and **Scott Morrison**, *The Karoubi Envelope and Lee's Degeneration of Khovanov Homology*, Algebraic & Geometric Topology, 6 (2006) pp. 1459–1469. Available at arXiv:math.GT/0606542. [A]

**(2)**

I have been awarded a Discovery Early Career Researcher Award, with project ID DE120100232. Its title is "Fusion categories and topological quantum field theory". The project has now been active for six months. The funded amount is $375,000. I have completed three preprints (all available at arxiv.org), which are described in §H (Statements of Progress).

| Project Id | CI Name | Amount Funded | Years | Title | Publications |
|---|---|---|---|---|---|
| DE120100232 | Dr S Morrison | $375,000 | 3 | Fusion categories and topological quantum field theory | [M:1], [M:18] & [M:19] in §H |

**F15.3. Ten career-best publications**

(Please attach a PDF with a list of your ten career-best publications (10 pages maximum). Provide the full reference for each of your ten best publications. Next to each provide information on any ARC grant scheme on which you were a Chief Investigator from which they originated, as described in F15.2. Add a statement of a maximum of 30 words explaining and justifying the impact or significance of each publication. Asterisk the publications relevant to this Proposal.)

## F15.3. Ten career-best publications (Scott Morrison)

* **(1)** Masaki Izumi, Vaughan Jones, **Scott Morrison**, and Noah Snyder, *Subfactors of index less than 5, part 3: quadruple points* [M:2] [J:2]. Communications in Mathematical Physics, vol. 316, issue 2 (2012), pp. 531–554, available at arXiv:1109.3190. (8 citations)

Close analysis of connections of certain principal graphs with quadruple points enables us to complete the classification of subfactors with index less than 5.

* **(2)** **Scott Morrison**, David Penneys, Emily Peters and Noah Snyder, *Subfactors of index less than 5, part 2: triple points* [M:3]. International Journal of Mathematics, vol. 23, No. 3 (2012) 1250016, available at arXiv:1007.2240. (13 citations)

Applying a wide variety of techniques (connections, graph planar algebras, and quadratic tangles), we deal with cases of the classification of subfactors with index less than 5 involving triple points.

* **(3)** **Scott Morrison** and Noah Snyder, *Subfactors of index less than 5, part 1: the principal graph odometer* [M:4]. Communications in Mathematical Physics, vol. 312, issue 1 (2012), pp. 135, available at arXiv:1007.1730. (17 citations)

We announce the classification of subfactors with index less than 5, delegating parts of the proof to later papers in the series.

**(4)** **Scott Morrison** and Kevin Walker, *Higher categories, colimits and the blob complex* [M:5], Proceedings of the National Academy of Sciences, May 2 2011, available at. (3 citation)

Building on our paper 'the blob complex', we present the core results in one of the highest impact journals, and give a newer, simpler description of the main construction.

**(5)** **Scott Morrison** and Kevin Walker, *The blob complex* [M:6]. Geometry & Topology, 16 (2012) 1481–1607, available at arXiv:1009.5025. (11 citations)

This 106 page paper at the top topology journal introduces a generalization of topological field theory, incorporating homotopy theory. We introduce disk-like higher categories and prove a higher Deligne conjecture.

* **(6)** Frank Calegari, **Scott Morrison** and Noah Snyder, *Cyclotomic integers, fusion categories, and subfactors* [M:7], Communications in Mathematical Physics Vol. 303, Issue 3 (2011), pp. 845–896. Available at arXiv:1004.0665. (8 citations)

We proved that in certain infinite families of graphs, at most finitely many have squared graph norm which is cyclotomic. This is now an essential tool in subfactor classification problems.

* **(7)** **Scott Morrison** and Noah Snyder, *Non-cyclotomic fusion categories* [M:9], to appear in Transactions of the American Mathematical Society (accepted 3 Nov 2010), available at arXiv:1002.0168. (4 citations)

We answered in the negative a question of Etingof, Nikshych and Ostrik: "Are all fusion categories defined over a cyclotomic field?", using our new construction of certain exotic examples.

* **(8)** Stephen Bigelow, **Scott Morrison**, Emily Peters and Noah Snyder, *Constructing the extended Haagerup planar algebra* [M:10], Acta Mathematica, vol. 209 (2012), pp. 29–82, available at arXiv:0909.4099. (28 citations)

This paper solved a longstanding open problem, constructing an exotic subfactor. We introduced powerful new techniques for planar algebras, and completed the classification of subfactors with index less than $3 + \sqrt{3}$.

**(9)** Michael Freedman, Robert Gompf, **Scott Morrison** and Kevin Walker, *Man and machine thinking about the smooth 4-d Poincaré conjecture* [M:12], Quantum Topology, Vol. 1, Issue 2 (2010), pp. 171–208. Available at `arXiv:0906.5177`. (21 citations)

We propose a new approach to finding obstructions which might disprove the smooth 4-dimensional Poincaré conjecture, and try the first potential counterexample.

**(10)** David Clark, **Scott Morrison** and Kevin Walker, *Fixing the functoriality of Khovanov homology* [M:14], Geometry & Topology, 13 (2009) pp. 1499–1582. Available at `arXiv:math.GT/0701339`. (36 citations)

This paper at the top topology journal defines a new version of Khovanov homology (a powerful categorical knot invariant) which is functorial.

**F15.4. Further evidence in relation to research impact and contributions to the field over the last 10 years.**

(Write a maximum of 7500 characters (approx 1000 words). In this section, provide: (1) Research outputs other than publications. Other research outputs might include patents and policy advice, competitive grants and other research support, relevant consultancies, and other professional activities or other outputs; and (2) Evidence for the quality of your research outputs including those in F15.2 to F15.4. Assess the impact of your research for all of your outputs relative to opportunity and in the context of discipline expectations. Include a wide range of research evaluations of impact (for example, citations, evaluations of the publication's quality; the journal, the book publishing house, the conference etc; and any other measures of impact; honours and awards/prizes, esteem measures, and any other evaluations of your outputs).)

I am the organiser of several international conferences, including:
* a workshop on subfactors and von Neumann algebras to be held at the Banff International Research Station in 2014,
* a workshop on fusion categories and subfactors, (sponsored by the American Institute of Mathematics, held in March 2012),
* two conferences on subfactors, (sponsored by the Defence Advanced Research Projects Agency, held in July 2011 and July 2012), and
* a session of a joint AMS/NZMS meeting (in Wellington, 2007).

I have refereed over 20 papers in my fields, including many for prestigious journals such as Geometry & Topology and Geometric and Functional Analysis (both ERA A*).

I am a founding moderator of MathOverflow, a new internet site for questions and answers on research level mathematics. The site sees over 10,000 visits each day, from the strongest undergraduate and postgraduate students in mathematics, to many of the international research leaders in the field. Now in its fourth year, MathOverflow is revolutionising mathematical research, by initiating collaborations, integrating mathematical communities, and providing access to the highest calibre expertise in specialised fields. (See my article in the Notices of the American Mathematical Society for more details.)

I actively blog on the Secret Blogging Seminar, founded by 7 other UC Berkeley PhDs. We've been active for 5 years, covering research, pedagogy, and mathematical news. I recently launched Math2.0, a forum for discussing the future of mathematical publishing.

I created, with Dror Bar-Natan, the Knot Atlas, a widely used encyclopedia of knot theoretic data, which has spurred significant research and been cited in over 60 peer-reviewed mathematical papers.

I maintain research links with mathematicians working in industry, particularly at Microsoft Station Q (a research center on topological quantum computing). In joint work with David Spivak at MIT I have been developing a new mathematical model of databases, which has led to visits with the informatics group at Amgen, a large pharmaceuticals company.

I received the Herb Alexander Prize for best mathematics thesis at UC Berkeley in 2007.

I have given invited talks at many international conferences, including:
* The search for small fusion categories
Plenary talk at the AustMS annual meeting
Ballarat, Australia, September 2012
* Khovanov homology for 4-manifolds and the blob complex
New Perspectives in Topological Field Theories
Hamburg, Germany, August 2012.
* Turaev-Viro theory and connections
Subfactors in Maui.
Hawaii, USA, July 2012.
* Constructing exotic subfactors
AIM workshop on 'Classifying small fusion categories'
Palo Alto, USA, March 2012.
* Can we get to 3+\sqrt{5}?
Kyoto conference on von Neumann algebras,
Kyoto, Japan, January 2012.

* Connections and planar algebras.
Von Neumann algebras and Conformal Field Theory,
Vanderbilt University, October 2011.
* Classifying fusion categories and subfactors.
SIAM Mini-symposium on Algebraic Aspects of Quantum Computing,
Raleigh, USA, October 2011.
* Classifying small index subfactors.
Great Plains Operator Theory Symposium,
Phoenix, USA, May 2011.
* Classification of subfactors up to index 5.
Quantum Groups,
Clermont-Ferrand, France, September 2010.
* Classification of subfactors up to index 5.
ICM Satellite Conference on Operator Algebras
Chennai, India, August 2010.
* The blob complex.
Low dimensional topology and categorification,
Stony Brook,USA, June 2010.
* Khovanov homology.
Link homology
Mathematical Sciences Research Institute, USA, January 2010.
* The blob complex.
TQFT and Link Homology
Hahei, New Zealand, January 2010.
* Fusion categories and small index subfactors.
Fusion categories
Waco, USA, October 2009.
* Extended Haagerup exists!
Tensor categories,
Bloomington, USA, March 2009.
* The Cappell-Shaneson spheres and the s-invariant.
Knots in Washington,
Washington DC, USA, January 2009.
* Blob homology.
Georgia Topology Conference,
Athens, USA, May 2008.
* Lasagna composition for Khovanov homology.
Joint NZMS/AMS meeting,
Wellington, New Zealand, December 2007.
* Lasagna composition for Khovanov homology.
Quantum Topology in Hanoi,
Vietnam, August 2007.
* Lasagna composition for Khovanov homology.
Link homology,
Oporto, Portugal, July 2007.
* Functoriality and duality for Khovanov homology.
Categorification,
Kyoto, Japan, May 2007.
* Fixing the functoriality of Khovanov homology.
Categorification in Algebra and Topology,
Uppsala, Sweden, September 2006.
* Khovanov homology.
Low dimensional topology,
Taipa, New Zealand, January 2006.

Mathematics department colloquiums:
* Small fusion categories, Vanderbilt, November 2012
* Knots and quantum computation, Wollongong, October 26 2012
* Khovanov homology and 4-manifold invariants, Eugene November 2011.
* Small index subfactors, Toronto, October 2011.
* Fusion categories and subfactors, UC San Diego, January 2011.
* Fusion categories and subfactors, Caltech, November 2010.

* Fusion categories, UC Berkeley, September 2009.
* Classifying subfactor planar algebras, UC Riverside, June 2009.

My papers appear in a variety of excellent international research journals. I have also published in the Proceedings of the National Academy of Sciences; it is one of the most highly regarding general science journals, and publishes mathematics quite infrequently. I consider this achievement strong evidence for the broad impact of my research.

I include below some excerpts from referee reports on my papers:

The blob complex
(Geometry & Topology, ERA A*, MCQ 1.45, 11 citations) [M:6]

I think the paper is both substantial and significant. It touches on a variety
of subjects both in pure and physically motivated mathematics and relates
them in a non-trivial way. ... Overall, this is a very interesting paper, and
I believe that Geometry & Topology would be a suitable journal for it.

Constructing the extended Haagerup planar algebra
(Acta Mathematica, ERA A*, MCQ 3.53, 28 citations) [M:10]:

They combine very skillfully brand new techniques from Jones's theory of
planar algebras, with an amazing new idea, the so-called jellyfish algorithm.
... The paper is extremely well written, the result is first rate, the proof is
clever and original. In my view, the paper deserves to be published in
Acta Mathematica.

Subfactors of index less than 5, part 1
(Communications in Mathematical Physics, ERA A*, MCQ 1.38, 17 citations) [M:4]:

This classification is a major achievement in subfactor theory and
I am sure that it will attract attention of many mathematicians and
physicists for many years to come, so I am happy to recommend
publication of this paper in [Communications in Mathematical Physics].

Cyclotomic integers, fusion categories, and subfactors
(Communications in Mathematical Physics, ERA A*, MCQ 1.38, 8 citations) [M:7]:

... the results are of great interest in today's research in mathematical
physics and obviously deserve publication in Commun. Math. Phys.

**F15.5. A statement on your most significant contributions to this research field of this Proposal.**

(Write a maximum of 3750 characters (approx 500 words).)

Numbered references are to my published work in F15.2. Other references are in C.

My research fields are fusion categories, subfactors, higher category theory and low dimensional topology. I have made significant contributions to each of these fields. This proposal is focused on subfactors and their representation theory, and builds upon my work there, yet will also draw on ideas from my work in higher category theory.

Subfactors:

I have demonstrated expertise in the study of subfactors, exemplified by my paper 'Constructing the extended Haagerup planar algebra' [9] in Acta Mathematica.

This paper answers a long-standing open problem in the classification of finite depth subfactors (c.f. section C for background). The extended Haagerup subfactor had been suspected to exist for 15 years, but had eluded all efforts to construct it. My paper introduced new techniques in quantum topology and skein theory, and applied these to a problem in operator algebras and subfactors.

My solution to this problem completed the classification of subfactors with index less than 3+\sqrt{3}.

I then lead an international collaboration to extend the classification program up to index 5. We have now established this result, showing that there are exactly 10 finite depth subfactors with index between 4 and 5. This work has been published in a four part series; Parts [1] and [3] appeared in Communications in Mathematical Physics and Part [2] at the International Journal of Mathematics. Part 4 [PT10] was written by graduate students at UC Berkeley following the techniques of my paper [6]. This series includes a total of 7 coauthors, and this broad collaboration will provide part of the international network supporting my proposed future work.

My paper 'Cyclotomic integers, fusion categories and subfactors' [6], in Communications in Mathematical Physics, introduces techniques from number theory to study subfactors. The paper provides a new approach to restricting the possible indices of subfactors, and also provides a powerful technical tool in classification results. These news tools seem likely to solve a number of open problems in the classification of subfactors, and will be essential for the proposed project.

Fusion categories:

In my paper 'Non-cyclotomic fusion categories' [8] in Transactions of the American Mathematical Society, I showed that the fusion categories coming from the Haagerup subfactor [AH99] and the newly constructed extended Haagerup subfactor [9] are exotic, in the sense that they can not be defined over any cyclotomic field. This answered a conjecture in literature [ENO05] in the negative; these were the first known non-cyclotomic fusion categories. My number theoretic work [6] mentioned above also had applications to the possible dimensions of objects in fusion categories.

Higher category theory and low dimensional topology:

I have a second major research project, on higher category theory and generalisations of Hochschild homology to arbitrary manifolds. My paper [4] describing the first results obtained in this project appeared in the Proceedings of the National Academy of Sciences. A second paper [5] on this work has been published at Geometry & Topology. The deeper understanding of general topological quantum field theories obtained through that work will inform and motivate the work described in the project.

## PART F - Personnel (Dr Pinhas Grossman)

### F1. Personal details

(The personal details will be filled out for you automatically. To update any of your personal details in this form, please update your profile accordingly and your details will update automatically in this form.)

**Title**

Doctor

**Family Name**

Grossman

**First Name**

Pinhas

**Person identifier**

03419273

**Role**

Chief Investigator

### F2. Postal address

(The postal address will be filled out for you automatically. To update your postal address, please update your profile accordingly and your postal address will update automatically in this form.)

**Postal Address Line 1**

School of Mathematics and Statistics

**Postal Address Line 2**

University of New South Wales

**Locality**

Sydney

**State**

NSW

**Postcode**

2052

**Country**

| |
|---|
| Australia |

**F3. Are you a current member of the ARC or its selection or other advisory committees?**

(This relates only to ARC College or Selection Advisory Committee members for National Competitive Grants Program funding schemes.)

**Current Member of Advisory Committee**

| |
|---|
| No |

**F4. Please name any of your relatives or close social/professional associates that are members of the ARC or its selection or other advisory committees.**

| | Title | First Name | Family Name |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

**F5. Please name any Commonwealth-funded Research Centres that you will be associated with as at 1 January 2014.**

| | Full Legal Name of Centre | Start Date | Cessation Date | Centre Role |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |

| | Centre Role if Other |
|---|---|
| 1 | |
| 2 | |

**F6. Are you an Indigenous Participant?**

**Indigenous Participant**

| |
|---|
| No |

**F7. Awarded ARC Fellowships and ARC Awards**

(Please provide the name and the funding period for any awarded ARC Fellowship(s) or ARC Award(s).
(For example: ARF 2005-2009, DORA 2012-2014))

| |
|---|
| |

**F8. PhD Qualification**

**F8.1. Do you hold a PhD or expect to be awarded a PhD qualification in the near future?**

**PhD Yes/No**

| |
|---|
| Yes |

**F8.2. If you hold a PhD or expect to be awarded a PhD qualification in the near future, please enter the date your PhD has been awarded or the date your thesis will be submitted, respectively.**

**Date of Award**

19/05/2006

**F9. Qualifications**

| | Degree/Award | Year | Discipline/Field | Organisation Name |
|---|---|---|---|---|
| 1 | PhD | 2006 | Mathematics | University of California, Berkeley |

| | Country |
|---|---|
| 1 | United States of America |

**F10. Current and previous appointment(s)/position(s) – during the past 10 years**

| | Position | Organisation Name | Department | Year Appointed |
|---|---|---|---|---|
| 1 | Lecturer | The University of New South Wales | Pure Mathematics, School of Mathematics and statistics | 2013 |
| 2 | Postdoctoral Fellow | IMPA | IMPA - Institute of Pure and Applied Mathematics , Brazil | 2010 |
| 3 | Marie Curie Research Fellow | Cardiff University | School of Mathematics | 2009 |
| 4 | Assistant Professor (NTT) and Postdoctoral Fellow | Vanderbilt University | Mathematics | 2006 |

| | Continuity | Employment Kind | Current |
|---|---|---|---|
| 1 | Permanent | Full Time | Yes |
| 2 | Contract | Full Time | No |
| 3 | Contract | Full Time | No |
| 4 | Contract | Full Time | No |

**F11. Organisational affiliations for eligibility purposes for this Proposal**

(Name of the organisation you will be associated with for the purposes of satisfying the eligibility requirements for your nominated role in undertaking the proposed research (i.e. for a CI this will usually be the Eligible Organisation at which they will employed or hold an adjunct appointment as at 1 January 2014 and beyond; for PIs it will generally be their main employer as at 1 January 2014).)

**Organisation Name**

The University of New South Wales

**Type of Affiliation**

Employee

**F12. What is your time commitment (%FTE) to this Project?**

40

**F13. Are you requesting an International Collaboration Award?**

(Note: If you are an Australian-based PI, you must choose 'No'. Also, if you are a PI working in an Australian Eligible Organisation overseas campus you must choose 'No'.)

**International Collaboration Award**

| |
|---|
| Yes |

**F14. Discovery Outstanding Researcher Award**

**F14.1. Justification**

(In no more than two A4 pages please justify how this Discovery Outstanding Researcher Award would benefit, enhance and expedite the overall Project.)

**Justification**

| |
|---|
| PDF attachment not submitted |

**F15. Research Record Relative to Opportunities**

**F15.1. Details on your career and opportunities for research over the last 10 years.**

(Write a maximum of 5250 characters (approx. 750 words).

Provide and explain:
(i) The number of years it has been since you graduated with your highest educational qualification;
(ii) The research opportunities that you have had in the context of your employment situation, the research component of your employment conditions, and any unemployment or part-time employment you may have had;
(iii) Whether you are a research-only, teaching and research, teaching only, teaching and administration, research and administration, or administration–only academic, as well as giving any additional information (for example, part-time status) needed to understand your situation. Give an indication of what percentage of time you have spent over the last ten years in those roles;
(iv) Any career interruptions you have had for childbirth, carer's responsibility, misadventure, or debilitating illness;
(v) The research mentoring and research facilities available to you; and
(vi) Any other aspects of your career or opportunities for research that are relevant to assessment and that have not been detailed elsewhere in this Proposal (for example, any circumstances that may have affected the time you have had to conduct and publish research).)

Since receiving my PhD in 2006 from the University of California at Berkeley, I have held postdoctoral research positions on 3 different continents before coming to Australia, allowing me build professional relationships with a large network of international researchers, with whom I maintain connections.

* Vanderbilt University, September 2006 - August 2009
I was a Research Training Group Postdoctoral fellow at Vanderbilt University, a position funded by the National Science Foundation. There I worked with the group in Noncommutative Geometry and Operator Algebras. I was also the principal investigator of a National Science Foundation grant awarded in 2008 for "Intermediate subfactors and planar algebras."

* Cardiff University, September 2009 - August 2010
In 2009-2010 I was a Marie-Curie Research Fellow at the Cardiff University node of the EU network in Noncommutative Geometry in Physics. The EU NCG network had 11 nodes in different European countries. I participated in various network activities and had the opportunity to liase with a wide spectrum of the European research community in both mathematics and physics.

* Institute of Pure and Applied Mathematics (IMPA), September 2010 - December 2012
From 2010-2012 I was a postdoctoral fellow at the Institute of Pure and Applied Mathematics (IMPA) in Rio de Janeiro, Brazil, as part of the new special postdoctoral program. While at IMPA I spoke at both

conferences and seminars at various Brazilian universities, while maintaining my international collaborations in the US and elsewhere.

* University of New South Wales
Beginning 2013, I have been appointed to a continuing teaching/research position as a lecturer (Level B) in Pure Mathematics at the School of Mathematics and Statistics at UNSW. The Department of Pure Mathematics has a strong and active research group in modern analysis and operator theory, which are core areas of this project. This group includes several senior mathematicians who I expect will mentor me over the course of the project and throughout the coming years.

During the past several years, I have written 8 papers, 6 of which have already been published in leading international journals (see Section C for publication details.) I have also been invited to speak at numerous international conferences in many different countries, including the US, the UK, Canada, Brazil, France, Malaysia, and Denmark, among others, and have given colloquium and seminar lectures at many universities across the world.

From 2006-2008 I organized the Research Training Group seminar at Vanderbilt University, in which graduate students gave introductory talks on research topics in operator algebras and non-commutative geometry.

I have refereed papers for several major international research journals, including Advances in Mathematics, International Journal of Mathematics, Bulletin of the London Mathematical Society, Indiana University Mathematics Journal, and Mathematica Scandinavica.

**F15.2. Recent significant publications (since 2003)**

(Please attach a PDF with a list of your recent significant publications (40 pages maximum). (1) Provide your research publications published in the last ten years split into the five categories of (a) scholarly books, (b) scholarly book chapters, (c) refereed journal articles, (d) refereed conference papers only when the paper was published in full in the proceedings and, (e) other (for example, major exhibitions, compositions or performances). You must number your publications continuously. Asterisk the publications relevant to this Proposal. (2) Provide a list of your ARC grants awarded in the last ten years on which you have been a Chief Investigator. Give the ARC grant number, Chief Investigator names in the order that they appear on the grant, the amount funded, the years for which the grant was awarded, and the title of the grant. Please refer to the Instructions to Applicants for format requirements. With respect to your numbered publications in the last ten years given in part 1 of question F15.2, next to each ARC grant, provide the numbers of the publications from part 1 of question F15.2 that arose from, or were in part supported by, your ARC grants.)

## F15.2. Recent significant publications (Pinhas Grossman)

**(1)**

**Refereed journal articles**

*At the end of each line, the 2010 ERA ranking and 2011 Mathematical Citation Quotient are shown in square brackets. The overall MCQ for 2011 is 0.32; notice that every one of my publications is in a journal with a higher MCQ, and many are significantly higher.*

* **[G:1]** **Pinhas Grossman** and Noah Snyder, *Quantum subgroups of the Haagerup fusion categories*. Communications in Mathematical Physics, vol. 311 (2012), pp. 617-643. [A*, 1.38]

* **[G:2]** Marta Asaeda and **Pinhas Grossman**, *A quadrilateral in the Asaeda-Haagerup category*. Journal of Quantum Topology, vol. 2 (2011), pp. 269-300. [new journal - unranked, 1.54]

* **[G:3]** **Pinhas Grossman** and Alan Wiggins, *Strong singularity for subfactors*. Bulletin of the London Mathematical Society, vol. 42 (2010), pp. 607-620. [A, 0.52]

* **[G:4]** **Pinhas Grossman** and Masaki Izumi, *Classification of noncommuting quadrilaterals of factors*. International Journal of Mathematics, vol. 19 (2008), pp. 557-643. [A , 0.52]

* **[G:5]** **Pinhas Grossman**, *Forked Temperley-Lieb algebras and intermediate subfactors*. Journal of Functional Analysis, vol. 247 (2007), pp. 477-491. [A*, 1.13]

* **[G:6]** **Pinhas Grossman** and Vaughan F. R. Jones, *Intermediate subfactors with no extra structure*. Journal of the American Mathematical Society, vol. 20 (2007), pp. 219-265. [A*, 3.13]

**(2)**

No previous ARC grants.

### F15.3. Ten career-best publications

(Please attach a PDF with a list of your ten career-best publications (10 pages maximum). Provide the full reference for each of your ten best publications. Next to each provide information on any ARC grant scheme on which you were a Chief Investigator from which they originated, as described in F15.2. Add a statement of a maximum of 30 words explaining and justifying the impact or significance of each publication. Asterisk the publications relevant to this Proposal.)

## F15.3. Ten career-best publications (Pinhas Grossman)

* **(1)** **Pinhas Grossman** and Noah Snyder, *Quantum subgroups of the Haagerup fusion categories*. Communications in Mathematical Physics, vol. 311 (2012), pp. 617-643.

  We classify the quantum subgroups of the Haagerup fusion categories, providing one of the first such classifications for any interesting example. The techniques we develop are applicable to many other examples.

* **(2)** Marta Asaeda and **Pinhas Grossman**, *A quadrilateral in the Asaeda-Haagerup category*. Journal of Quantum Topology, vol. 2 (2011), pp. 269-300. [new journal - unranked, 1.54]

  We construct a new exotic subfactor in the Morita family of the Asaeda-Haagerup subfactor. As part of the construction we illustrate how to perform computations with the diagrammatic calculus for tensor categories as applied to connections on graphs.

* **(3)** **Pinhas Grossman** and Alan Wiggins, *Strong singularity for subfactors*. Bulletin of the London Mathematical Society, vol. 42 (2010), pp. 607-620. [A, 0.52 ]

  We study the notion of strong singularity, previously only considered for MASA's, in the context of subfactors. We give an example of a subfactor which is singular but not 1-strongly singular, which does not occur in the MASA case.

* **(4)** **Pinhas Grossman** and Masaki Izumi, *Classification of noncommuting quadrilaterals of factors*. International Journal of Mathematics, vol. 19 (2008), pp. 557-643.

  We provide a wide-ranging classification of quadrilaterals of subfactors into several classes and classify all quadrilaterals of small index. We also construct various new examples of subfactors and quadrilaterals.

* **(5)** **Pinhas Grossman**, *Forked Temperley-Lieb algebras and intermediate subfactors*. Journal of Functional Analysis, vol. 247 (2007), pp. 477-491.

  We classify quadrilaterals of subfactors related to the Temperley-Lieb algebras by algebraic invariants.

* **(6)** **Pinhas Grossman** and Vaughan Jones, *Intermediate subfactors with no extra structure*. Journal of the American Mathematical Society, vol. 20 (2007), pp. 219-265.

  We classify quadrilaterals of subfactors with no extra structure, showing that such quadrilaterals exhibit a strong form of rigidity, analogous to the quantization of index for subfactors. A particularly surprising result is that when all four sides of the quadrilateral have no extra structure, only two examples exist.

**F15.4. Further evidence in relation to research impact and contributions to the field over the last 10 years.**

(Write a maximum of 7500 characters (approx 1000 words). In this section, provide: (1) Research outputs other than publications. Other research outputs might include patents and policy advice, competitive grants and other research support, relevant consultancies, and other professional activities or other outputs; and (2) Evidence for the quality of your research outputs including those in F15.2 to F15.4. Assess the impact of your research for all of your outputs relative to opportunity and in the context of discipline expectations. Include a wide range of research evaluations of impact (for example, citations, evaluations of the publication's quality; the journal, the book publishing house, the conference etc; and any other measures of impact; honours and awards/prizes, esteem measures, and any other evaluations of your outputs).)

I received the Herb Alexander prize from UC Berkeley in 2006 for "an outstanding dissertation in pure mathematics."

Since then I have conducted research in the areas of subfactors and fusion categories, publishing the results of my work in internationally recognized peer-reviewed journals. In addition, I have been actively involved in the international community, having held academic positions on four different continents and delivered invited lectures at numerous international research conferences and seminars.

I have given invited research talks at many international conferences, including:

* Mathematical Physics in Bahia, July 2012
* Subfactors in Maui, July 2012
* Jairo Charris Seminar: Categories, Geometry, and Physics, Universidad Sergio Arboleda, August 2011
* Maui Subfactor Conference, Maui, July 2011
* Brazilian Operator Algebras Symposium, Florianopolis, February 2011
* Quantum Groups and Noncommutative Geometry and Operator Algebras, CIRM, September 2010
* Eighth annual Spring Institute on Noncommutative Geometry and Operator Algebras, Vanderbilt University, May 2010
* EU Noncommutative Geometry Network Third Annual Meeting, Cardiff University, June 2010
* Period on Planar Algebras and Physics, (lecture series), Cardiff University, February 2010
* EU Noncommutative Geometry Network Conference, University of Copenhagen, October 2009
* K-Theory, C*-algebras, and Topology of Manifolds, Chern Institute, Tianjin, China, June 2009
* Asian Mathematical Conference (Special Session on Operator Algebras), Kuala Lumpur, Malaysia, June 2009
* East Coast Operator Algebras Symposium, Penn State University, October 2008
* International Conference on Operator Theory, Timisoara, Romania, July 2008
* Great Plains Operator Theory Symposium, University of Cincinnati, June 2008 (plenary lecture)
* AMS Western Section Meeting, (Special Session on Subfactors and C*-algebras), Claremont McKenna College, May 2008
* Topics in von Neumann Algebras II, Banff International Research Station, March 2008
* Workshop on Free Probability, Random Matrices, and Planar Algebras, Fields Institute, Toronto, Canada, September 2007
* International Conference on Quantum Topology, Institute of Mathematics, Hanoi, Vietnam, August 2007
* Subfactors in Maui, Hawaii, July 2007
* Arbeitsgemeinschaft on Algebraic Structures in Conformal Field Theory, Mathematisches Forschungsinstitut Oberwolfach, Germany, April 2007
* East Coast Operator Algebras Symposium, Georgia Tech, October 2006
* Topics in von Neumann Algebras, Banff International Research Station, September 2006
* Brazilian Operator Algebras Conference, a satellite conference of the International Congress on Mathematical Physics, Florianopolis, Brazil, July 2006
* Third annual Spring Institute on Noncommutative Geometry and Operator Algebras, Vanderbilt University, May 2005

I have also given numerous research talks at university seminars and
colloquia, including:

* Federal University of Rio de Janeiro, October 2012
* Hunter College of the City University of New York, May 2012
* US Naval Academy, February 2012
* Queens University, January 2012

* Federal University of Santa Catarina, November 2011
* IMPA, Brazil, June 2012
* Cardiff University, June 2011
* Vanderbilt University, April 2011
* IMPA, Brazil, January 2011
* Katholieke Universiteit Leuven, April 2010
* Glasgow University, October 2009
* Cardiff University (series of talks) September – October 2009
* University of Rome, Tor Vergata, March 2009
* University of Houston, February 2009
* Wayne State University, February 2009
* SUNY at Buffalo, November 2008
* Texas A&M University, September 2008
* University of Tokyo, December 2007
* Vanderbilt University (various talks) 2006-2008
* University of Tokyo, Kyoto University and Kyushu University, Summer 2005.
* UC Berkeley (various talks) 2002-2006

I was an organizer of several workshops amd conferences:
* Shanks Workshop on Planar Algebras at Vanderbilt University in 2008
* Special Session on von Neumann algebras at the joint meetings of the American Mathematical Society / Mathematical Association of America in 2009
* LMS Regional Meeting and Workshop on Operator Algebras and Physics at Cardiff University in 2010.

**F15.5. A statement on your most significant contributions to this research field of this Proposal.**

(Write a maximum of 3750 characters (approx 500 words).)

My research areas are von Neumann algebras, subfactors, and fusion categories. I have made significant contributions in two major areas of research.

Subfactors

The classification of intermediate subfactors has been a longstanding open problem in subfactor theory. In the paper "Intermediate subfactors with no extra structure", with Vaughan Jones, published in the Journal of the American Mathematical Society, I made substantial progress on this problem by showing that intermediate subfactors are very rigid objects and their possible configurations are severely restricted.

This lead to a wide-ranging classification of quadrilaterals of subfactors, published in the paper "Classification of non-commuting quadrilaterals of factors", with Masaki Izumi.

Motivated by the study of quadrilaterals of factors, I then constructed together with Marta Asaeda a subfactor with index one larger than the exotic Asaeda-Haagerup subfactor.

Fusion categories

Together with Noah Snyder, I investigated the representation theory of the exotic fusion categories of Asaeda and Haagerup and discovered many new fusion categories and module categories. In particular, we completely described the Brauer-Picard groupoid of the Haagerup subfactor; this result was published in the paper "Quantum subgroups of the Haagerup fusion categories", in Communications in Mathematical Physics.

We also described a full sub-groupoid of the Brauer-Picard groupoid of the Asaeda-Haagerup subfactor. This work included many explicit computations of module categories, as well as conjectures about the full groupoid. This work is still in preprint form but has been submitted for publication.

Together with David Jordan, we then studied graded extensions of fusion categories and constructed a fusion category with an object of the smallest known dimension outside of the series of examples associated to the Dynkin diagrams. This work has also been submitted for publication.

## PART F - Personnel (Prof Vaughan Jones)

### F1. Personal details

(The personal details will be filled out for you automatically. To update any of your personal details in this form, please update your profile accordingly and your details will update automatically in this form.)

**Title**

Professor

**Family Name**

Jones

**First Name**

Vaughan

**Person identifier**

92426965

**Role**

Partner Investigator

### F2. Postal address

(The postal address will be filled out for you automatically. To update your postal address, please update your profile accordingly and your postal address will update automatically in this form.)

**Postal Address Line 1**

Department of Mathematics - 1326 Stevenson Center

**Postal Address Line 2**

Vanderbilt University

**Locality**

Nashville

**State**

TN

**Postcode**

37240

**Country**

United States of America

**F3. Are you a current member of the ARC or its selection or other advisory committees?**

(This relates only to ARC College or Selection Advisory Committee members for National Competitive Grants Program funding schemes.)

**Current Member of Advisory Committee**

No

**F4. Please name any of your relatives or close social/professional associates that are members of the ARC or its selection or other advisory committees.**

| | Title | First Name | Family Name |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

**F5. Please name any Commonwealth-funded Research Centres that you will be associated with as at 1 January 2014.**

| | Full Legal Name of Centre | Start Date | Cessation Date | Centre Role |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |

| | Centre Role if Other |
|---|---|
| 1 | |
| 2 | |

**F6. Are you an Indigenous Participant?**

**Indigenous Participant**

No

**F7. Awarded ARC Fellowships and ARC Awards**

(Please provide the name and the funding period for any awarded ARC Fellowship(s) or ARC Award(s).
(For example: ARF 2005-2009, DORA 2012-2014))

**F8. PhD Qualification**

**F8.1. Do you hold a PhD or expect to be awarded a PhD qualification in the near future?**

**PhD Yes/No**

Yes

**F8.2. If you hold a PhD or expect to be awarded a PhD qualification in the near future, please enter the date your PhD has been awarded or the date your thesis will be submitted, respectively.**

**Date of Award**

00/00/1979

**F9. Qualifications**

| | Degree/Award | Year | Discipline/Field | Organisation Name |
|---|---|---|---|---|
| 1 | Docteur es Sciences | 1979 | Mathematics | University of Geneva |

| | Country |
|---|---|
| 1 | Switzerland |

**F10. Current and previous appointment(s)/position(s) – during the past 10 years**

| | Position | Organisation Name | Department | Year Appointed |
|---|---|---|---|---|
| 1 | Distinguished Professor | Vanderbilt University | Mathematics | 2011 |
| 2 | Professor | University of California, Berkeley | Mathematics | 1985 |

| | Continuity | Employment Kind | Current |
|---|---|---|---|
| 1 | Permanent | Full Time | Yes |
| 2 | Permanent | Full Time | Yes |

**F11. Organisational affiliations for eligibility purposes for this Proposal**

(Name of the organisation you will be associated with for the purposes of satisfying the eligibility requirements for your nominated role in undertaking the proposed research (i.e. for a CI this will usually be the Eligible Organisation at which they will employed or hold an adjunct appointment as at 1 January 2014 and beyond; for PIs it will generally be their main employer as at 1 January 2014).)

**Organisation Name**

Vanderbilt University

**Type of Affiliation**

Employee

**F12. What is your time commitment (%FTE) to this Project?**

10

**F13. Are you requesting an International Collaboration Award?**

(Note: If you are an Australian-based PI, you must choose 'No'. Also, if you are a PI working in an Australian Eligible Organisation overseas campus you must choose 'No'.)

**International Collaboration Award**

No

## F14. Discovery Outstanding Researcher Award

### F14.1. Justification

(In no more than two A4 pages please justify how this Discovery Outstanding Researcher Award would benefit, enhance and expedite the overall Project.)

**Justification**

PDF attachment not submitted

## F15. Research Record Relative to Opportunities

### F15.1. Details on your career and opportunities for research over the last 10 years.

(Write a maximum of 5250 characters (approx. 750 words).

Provide and explain:
(i) The number of years it has been since you graduated with your highest educational qualification;
(ii) The research opportunities that you have had in the context of your employment situation, the research component of your employment conditions, and any unemployment or part-time employment you may have had;
(iii) Whether you are a research-only, teaching and research, teaching only, teaching and administration, research and administration, or administration–only academic, as well as giving any additional information (for example, part-time status) needed to understand your situation. Give an indication of what percentage of time you have spent over the last ten years in those roles;
(iv) Any career interruptions you have had for childbirth, carer’s responsibility, misadventure, or debilitating illness;
(v) The research mentoring and research facilities available to you; and
(vi) Any other aspects of your career or opportunities for research that are relevant to assessment and that have not been detailed elsewhere in this Proposal (for example, any circumstances that may have affected the time you have had to conduct and publish research).)

I received my degree of Docteur es Sciences (Mathematique) from the University of Geneva in 1979.

From 1985-- I have been a Professor of Mathematics at the University of California at Berkeley. In 2011 I took a position as Distinguished Professor at Vanderbilt University. My position is 75% research, 25% teaching.

During my research career I have published 95 papers, on subjects across operator algebras, knot theory, quantum computation, and others. Many of these papers have appeared in top research journals, including Annals of Mathematics, Acta Mathematica, Inventiones, and the Journal of the American Mathematical Society.

I have substantial mentoring experience, having supervised 24 Ph.D. students, and I currently have 3 more Ph.D. students under my supervision.

In 1990 I received the Fields Medal for my foundational work on subfactors, and the discovery of a knot polynomial deriving from that work.

I am a co-director of the New Zealand Mathematics Research Institute.

**F15.2. Recent significant publications (since 2003)**

(Please attach a PDF with a list of your recent significant publications (40 pages maximum). (1) Provide your research publications published in the last ten years split into the five categories of (a) scholarly books, (b) scholarly book chapters, (c) refereed journal articles, (d) refereed conference papers only when the paper was published in full in the proceedings and, (e) other (for example, major exhibitions, compositions or performances). You must number your publications continuously. Asterisk the publications relevant to this Proposal. (2) Provide a list of your ARC grants awarded in the last ten years on which you have been a Chief Investigator. Give the ARC grant number, Chief Investigator names in the order that they appear on the grant, the amount funded, the years for which the grant was awarded, and the title of the grant. Please refer to the Instructions to Applicants for format requirements. With respect to your numbered publications in the last ten years given in part 1 of question F15.2, next to each ARC grant, provide the numbers of the publications from part 1 of question F15.2 that arose from, or were in part supported by, your ARC grants.)

## F15.2. Recent significant publications (Vaughan F. R. Jones)

**(1)**

**Refereed journal articles**

*Articles with direct relevance to the proposed project are marked with a *. At the end of each line, the 2010 ERA ranking and 2011 Mathematical Citation Quotient are shown in square brackets. The overall MCQ for 2011 is 0.32; notice that most of my publications are in journals with significantly higher MCQs.*


* **[J:1]** **Vaughan F. R. Jones**, *Quadratic tangles in planar algebras.* Duke Math. J. 161 (2012), no. 12, 2257–2295. [A*, 1.76]

* **[J:2]** Masaki Izumi, **Vaughan F. R. Jones**, Scott Morrison, Noah Snyder, *Subfactors of index less than 5, part 3: quadruple points.* Comm. Math. Phys. 316 (2012), no. 2, 531554. [A*, 1.38]

* **[J:3]** Alice Guionnet, **Vaughan F. R. Jones**, Dimitri Shlyakhtenko, *A semi-finite algebra associated to a subfactor planar algebra.* J. Funct. Anal. 261 (2011), no. 5, 1345–1360. [A*, 1.12]

**[J:4]** **Vaughan F. R. Jones**, Dimitri Shlyakhtenko, Kevin Walker, *An orthogonal approach to the subfactor of a planar algebra.* Pacific J. Math. 246 (2010), no. 1, 187197. [A, 0.58]

* **[J:5]** Stephen Curran, **Vaughan F. R. Jones**, Dimitri Shlyakhtenko, *On the symmetric enveloping algebra of planar algebra subfactors*, accepted Trans. Amer. Math, 3 July 2012. Soc. [A*, 1.05]

* **[J:6]** **Vaughan F. R. Jones**, David Penneys, *The embedding theorem for finite depth subfactor planar algebras.* Quantum Topol. 2 (2011), no. 3, 301–337. [unranked, 1.54]

* **[J:7]** **Vaughan F. R. Jones**, *On the origin and development of subfactors and quantum topology.* Bull. Amer. Math. Soc. (N.S.) 46 (2009), no. 2, 309–326. [A*, 1.27]

* **[J:8]** **Vaughan F. R. Jones**, *Two subfactors and the algebraic decomposition of bimodules over $II_1$ factors.* Acta Math. Vietnam. 33 (2008), no. 3, 209–218. [A*, 3.53]

* **[J:9]** Pinhas Grossman, **Vaughan F. R. Jones**, *Intermediate Subfactors with No Extra Structure* J. Amer. Math. Soc. 20 (2007), no. 1, 219–265. [A*, 3.13]

* **[J:10]** **Vaughan F. R. Jones**, Sarah Reznikoff, *Hilbert space representations of the annular Temperley-Lieb algebra.* Pacific J. Math. 228 (2006), no. 2, 219–249. [A, 0.58]

* **[J:11]** **Vaughan F. R. Jones**, Feng Xu, *Intersections of finite families of finite index subfactors* Internat. J. Math. 15 (2004), no. 7, 717–733. [A, 0.51]

**[J:12]** Dorit Aharonov, **Vaughan F. R. Jones**, Zeph Landau, *A polynomial quantum algorithm for approximating the Jones polynomial.* Algorithmica 55 (2009), no. 3, 395421. [A*, 0.29]

**[J:13]** Marston Conder, **Vaughan F. R. Jones**, *Highly transitive imprimitivities.* J. Algebra 300 (2006), no. 1, 4456. [A*, 0.66]

* **[J:14]** Dietmar Bisch, **Vaughan F. R. Jones**, *Singly generated planar algebras of small dimension. II.* Adv. Math. 175 (2003), no. 2, 297318. [A*, 1.40]

**Refereed conference proceedings**

* [**J:15**] Alice Guionnet, **Vaughan F. R. Jones**, Dimitri Shlyakhtenko. *Random matrices, free probability, planar algebras and subfactors.* Quanta of maths, 201–239, Clay Math. Proc., 11, Amer. Math. Soc., Providence, RI, 2010.

[**J:16**] Dorit, Aharonov, **Vaughan F. R. Jones**, Zeph Landau, *A polynomial quantum algorithm for approximating the Jones polynomial*. STOC'06: Proceedings of the 38th Annual ACM Symposium on Theory of Computing, 427436, ACM, New York, 2006.

**(2)**

No previous ARC grants.

### F15.3. Ten career-best publications

(Please attach a PDF with a list of your ten career-best publications (10 pages maximum). Provide the full reference for each of your ten best publications. Next to each provide information on any ARC grant scheme on which you were a Chief Investigator from which they originated, as described in F15.2. Add a statement of a maximum of 30 words explaining and justifying the impact or significance of each publication. Asterisk the publications relevant to this Proposal.)

### List of Career-Best Publications

PDF attachment not submitted

### F15.4. Further evidence in relation to research impact and contributions to the field over the last 10 years.

(Write a maximum of 7500 characters (approx 1000 words). In this section, provide: (1) Research outputs other than publications. Other research outputs might include patents and policy advice, competitive grants and other research support, relevant consultancies, and other professional activities or other outputs; and (2) Evidence for the quality of your research outputs including those in F15.2 to F15.4. Assess the impact of your research for all of your outputs relative to opportunity and in the context of discipline expectations. Include a wide range of research evaluations of impact (for example, citations, evaluations of the publication's quality; the journal, the book publishing house, the conference etc; and any other measures of impact; honours and awards/prizes, esteem measures, and any other evaluations of your outputs).)

I have received a number of prizes and awards, recognizing the impact of my research.
1973 Swiss Government Scholarship (for study in Switzerland)
1973 F.W.W. Rhodes Memorial Scholarship
1980 Vacheron Constantin Prize (for thesis, Univ. de Geneve)
1983 Alfred P. Sloan Research Fellowship
1986 Guggenheim Fellowship
1990 Fellow of the Royal Society
1990 Fields Medal
1991 New Zealand Government Science Medal (now Rutherford Medal)
1991 Honorary Fellow RSNZ
1992 Honorary vice President for life, International Guild of Knot Tyers
1992 Honorary D.Sc., University of Auckland
1992 Corresponding Member, Australian Academy of Sciences
1993 Elected to American Academy of Arts & Sciences
1993 Honorary D.Sc. University of Wales.
1999 Elected to US National Academy of Sciences
2000 Onsager medal of Trondheim University
2001 Elected as a foreign member to the Norwegian Royal Society of Letters and Sciences.
2002 DCNZM
2002 Elected Honorary Member of the London Mathematical Society.
2002 Doctorat Honoris Causa, Universite du Littoral, Cote d'Opale
2004 Elected vice president, American Mathematical Society.
2007 Prix Mondial Nessim Habif
2007 UC Berkeley Faculty research lecture.
2009 DCNZM "upgraded" to KNZM (Knight Companion of the NZ order of merit.)

I have served as an editor or associate editor for many journals, including
* Transactions of the American Mathematical Society
* Pacific Mathematics Journal
* Annals of Mathematics
* New Zealand Journal of Mathematics
* Advances in Mathematics
* Journal of Operator Theory
* Reviews in Mathematical Physics
* Russian Journal of Mathematical Physics
* Journal of Mathematical Chemistry
* Geometry and Topology

* L'Enseignement Mathematique

I served on the Scientific Advisory Boards of the Fields Institute of Mathematics, the Erwin Schrödinger Institute, the Mathematical Sciences Research Institute at Berkeley, the Center for Communications Research (United States), the Institut Henri Poincaré, and the Miller Institute for Basic Research in Sciences.

**F15.5. A statement on your most significant contributions to this research field of this Proposal.**

(Write a maximum of 3750 characters (approx 500 words).)

I initiated the study of subfactor theory, and discovered deep links between the seemingly disparate fields of operator algebras, low dimensional topology, and mathematical physics. This discovery had particularly profound implications for knot theory and led to the development of a new mathematical discipline that has come to be known as quantum topology.

From Joan Birman's introduction to my Fields Medal work:

"In 1984 Jones discovered an astonishing relationship between von Neumann algebras and geometric topology. As a result, he found a new polynomial invariant for knots and links in 3-space. His invariant had been missed completely by topologists, in spite of intense activity in closely related areas during the preceding 60 years, and it was a complete surprise. As time went on, it became clear that his discovery had to do in a bewildering variety of ways with widely separated areas of mathematics and physics .... These included (in addition to knots and links) that part of statistical mechanics having to do with exactly solvable models, the very new area of quantum groups, and also Dynkin diagrams and the representation theory of simple Lie algebras. The central connecting link in all this mathematics was a tower of nested algebras which Jones had discovered some years earlier in the course of proving a theorem which is known as the 'Index Theorem'."

More recently, I have introduced and developed the theory of planar algebras. Planar algebras are an algebraic formalism which can be used to study the standard invariant of a subfactor; they are also intimately related to knot theory and statistical mechanical lattice models. Planar algebras are now studied by a diverse array of mathematicians and have been successfully exploited by myself and others in the construction and classification of subfactors.

# PART G - Research Support (DP140100732)

## G1. Research support for all participants

(For each participant on this Proposal, provide details of research funding (ARC and other agencies in Australia and overseas) for the years 2012 to 2016 inclusive. That is, list all projects/proposals/awards/ fellowships awarded or requests submitted involving that Participant for funding. Please refer to the Instructions to Applicants for submission requirements.)

# G Research Support

| Description | Same Research Area | Support Status | Project ID | 2012 | 2013 | 2014 | 2015 | 2016 |
|---|---|---|---|---|---|---|---|---|
| V Jones, P Grossman, S Morrison, **Symmetries of subfactors** | Yes | R | DP140100732 | | | 132 | 139 | 134 |
| S Morrison, **Fusion categories and topological quantum field theory.** | Yes | C | DE120100232 | 62.5 | 125 | 125 | 62.5 | |
| D Bisch, V Jones, S Morrison, D Shlyakhtenko, K Walker, **Quantum symmetries: Planar Algebras and Free Probability** | Yes | P | HR0011-12-1-0009 | 228 | 228 | | | |

**Quantum symmetries: Planar Algebras and Free Probability** is a grant from DARPA (Defense Advanced Research Programs Agency).

**H1. For each participant on this Proposal, please attach a statement detailing progress for each project/fellowship involving that participant who has been awarded funding for 2012 under the ARC Discovery Projects, Discovery Indigenous, Linkage Projects or Fellowships scheme.**

| | Project ID | First named investigator | Scheme | Statement |
|---|---|---|---|---|
| 1 | DE120100232 | Scott Morrison | Discovery Early Career Researcher Award | |

## H Statements of Progress

### Scott Morrison DE120100232

My currently funded Discovery Early Career Researcher Award project "Fusion categories and topological quantum field theory", DE120100232 has been proceeding well through its first six months.

**Publications** I have completed one published article and three preprints developing the goals of the project:

[**M:1**] **Scott Morrison** and David Penneys, *Constructing spoke subfactors using the jellyfish algorithm* to appear in Transactions of the American Mathematical Society (accepted 19 Feb 2013), available at arXiv:208.3637. [A*, 1.05]

[**M:17**] **Scott Morrison**, *An obstruction to subfactor principal graphs from the graph planar algebra embedding theorem*, available at arXiv:1302.5148.

[**M:18**] **Scott Morrison** and Emily Peters, *The little desert? Some subfactors with index in the interval* $(5, 3+\sqrt{5})$, available at arXiv:1205.2742.

[**M:19**] Sabin Cautis, Joel Kamnitzer and **Scott Morrison**, *Webs and quantum skew Howe duality*, available at arXiv:1210.6437.

**Visitors** David Penneys (University of Toronto) visited for 4 weeks during January–February 2013 to collaborate on the project. Masaki Izumi (Kyoto University) will visit February 11–19 to discuss constructions of new subfactors, and extending techniques from subfactor theory so they are also applicable for fusion categories. Noah Snyder (University of Indiana) will visit March 10–18 to work on the classification of fusion categories with small global dimension, and to further investigate the Morita equivalence classes of the known exotic fusion categories. During April 27–May 12 Paramita Das and Shamindra Ghosh will visit from the Indian Statistical Institute in Kolkata.

**Travel** I have presented work from the DECRA project in talks at international research conferences and seminars, including:

- Northwestern geometry/physics seminar, Chicago, USA, November 16 2012.
- Vanderbilt colloquium, Nashville, USA, November 15 2012.
- New perspective in topological field theories, Hamburg, Germany, August 28 2012.
- Subfactors in Maui, Hawaii, USA, July 16 2012,

I was invited to give one of the plenary lectures at the Australian Mathematical Society meeting in Ballarat in 2012, where I spoke on 'the search for small fusion categories', summarizing the research goals and progress to date on the DECRA project.

Finally, I have travel planned to a number of international conferences to present results from the project and continue collaborations.

- Exceptional symmetries, Tokyo, Japan, March 4-8 2013. (I have been invited to give a series of lectures on the project.)
- Quantum topology, Nha Trang, Vietnam, May 13-17 2013.
- Miller Institute symposium, San Francisco, USA, July 7-8 2013.
- Mathematics and Quantum Physics, Rome, Italy, July 8-12 2013. (I have been invited as a plenary speaker.)
- Subfactors in Maui, Hawaii, USA, July 15-19 2013.

## I1. Other agencies

**Have you submitted or do you intend to submit a similar Proposal to any other agency?**

**Other Agency Submission**

No

**If Yes, please select one of the following:**

**Other Agency Name**

Not applicable for this candidate

**If Other is selected above, please enter the full name of the agency:**

Not applicable for this candidate

## I2. Does this proposal relate to any Indigenous Australian Societies?

**Indigenous Australian Society**

No

## I3. Please list internationally-based researchers who may be suitable to assess proposals in the same discipline area as this Proposal.

(Please list the name(s) in the following format: Title First Name Family Name, Organisation, Email Address.)

Masaki Izumi, Kyoto University, izumi@math.kyoto-u.ac.jp
Dietmar Bisch, Vanderbilt University, dietmar.bisch@vanderbilt.edu
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