Miller Institute Newsletter

Spring 2011

Miller Fellow Focus: Scott Morrison

Second year Miller Fellow Scott Morrison studies topological quantum field theories, and the algebraic data underlying them. One focus of his research is on the algebraic objects called 'subfactors' and 'fusion categories'. These objects are important in understanding topological quantum field theories in 3 dimensions, and are relevant in the study of topological phases of matter and also a certain approach to building quantum computers. His other focus is on the relationship between field theories and category theory in arbitrary dimension. This work has recently resulted in the introduction of the 'blob complex', which provides a homological extension of field theories. Scott Morrison is hosted by Prof. Vaughan Jones in the Department of Mathematics.

Topological quantum field theory is an important field of modern mathematics. Quantum Field Theory (QFT) attempts to describe the quantum mechanical evolution of fields (e.g. the electromagnetic field) on spacetime. The mathematics of QFT is extremely difficult, and there are significant untamed problems. Topological quantum field theory (TQFT) was originally introduced as a 'toy model' of a full quantum field theory. In TQFT, we assume that the evolution of fields does not depend on any geometric properties of spacetime (e.g. lengths, areas, durations and so on), but only on the topological shape of space-time. While this does not appear physically reasonable, the hope was that understanding the topological case would be easier, and eventually lead to insights towards a satisfactory mathematical formulation of QFT. While this program is still underway, significant progress has been made by mathematicians in understanding TQFT, and this understanding is now contributing to work on QFT. Furthermore, in the meantime physicists have discovered that the mathematicians' 'toy model' is physically relevant! The fractional quantum hall effect, one of the great discoveries of condensed matter physics in recent decades, has a (partial) mathematical description given by certain TQFTs in 3 dimensions. Moreover, materials exhibiting this behavior have been proposed as the hardware substrate for a quantum computer. This idea

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CALL FOR NOMINATIONS

Miller Fellowship nominations
due Thursday, September 8, 2011

Miller Professor applications
due Thursday, September 15, 2011

Visiting Miller Professor Departmental nominations
due Monday, September 19, 2011

Please see page 3 for details on making nominations for the Miller Fellowship program. For complete information on all our programs, visit: http://millerinstitute.berkeley.edu

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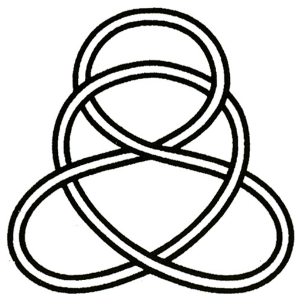
is being developed at Microsoft Station Q, where Scott worked prior to coming to the Miller Institute.

A topological quantum field theory in dimension $n + 1$ assigns a vector space to each "space", that is, a manifold of dimension n, and a number to each "spacetime", that is, a manifold of dimension $n + 1$. Essentially, this number is the quantum mechanical amplitude for the evolution of a space through a certain sequence of topological changes. An important property of TQFTs is that they are 'local': we can compute the number for a spacetime by decomposing it into smaller pieces, computing a certain invariant of each piece, and then assembling these answers by algebraic operations determined by the decomposition. In the end, everything is determined by a certain piece of algebraic data called an n-category. Conversely, given an n-category satisfying appropriate conditions, we can construct a TQFT in dimension $n + 1$.

When $n = 2$ (space is 2-dimensional while spacetime is 3), the algebraic data that determines a TQFT is called a fusion category. Recently Scott has been working on the classification of these objects. A complete classification is not feasible; indeed any finite group provides an example (its representation category). Instead he has been looking for a classification of small fusion categories. There are several good candidate notions of size for a fusion category. Each fusion category has a finite set of particle types (called simple objects by mathematicians), and the rank of a fusion category is the size of this set. Alternatively, there is a real number called the 'global dimension' associated to each fusion category. Recent progress by Etingof, Nikshych and Ostrik shows that when the global dimension is an integer and less than 84 the fusion category is 'weakly group theoretical' and essentially understood. When the global dimension is not an integer there are no strong results. Finally, each individual particle type has its own dimension, which is a real number. In quantum mechanics, a spin-½ particle has two states, and a quark has three 'colors', but in the wilder world of fusion categories the corresponding number need not be an integer. Scott's work recently has been on the classification of fusion categories containing an object with a small dimension. Dimensions up to 2 are well understood; there are particle types in fusion categories with dimensions of the form $2 \cos(\pi/n)$, as well as a variety of particle types with dimension exactly 2. One surprising consequence of Scott's new results is that the spectrum of possible dimensions remains discrete above 2. The next possible value is $(\sqrt{3} + \sqrt{7}) / 2$ and after that $\sqrt{5}$. Although examples are known at these dimensions, a complete description of particle types with these dimensions is not yet available.



Much of this classification has been obtained indirectly by first classifying subfactors with small index. A subfactor is an inclusion of von Neumann algebras each with trivial centre. Although the subject has its origins in analysis, subfactors and fusion categories are intimately related. Obtaining the classification results requires techniques from across a broad range of mathematical disciplines: representation theory, combinatorics, analysis, number theory and topology! This project has involved collaboration with David Penneys and James Tener, graduate students at Berkeley, and with Masaki Izumi (Kyoto, and a former Miller Fellow), Vaughan Jones (Berkeley), Emily Peters (MIT) and Noah Snyder (Columbia). A series of papers **Subfactors with index less than 5, parts 1-4** describes these results.

In a somewhat different direction, Scott has been working on extending the TQFT framework via a construction which he calls the 'blob complex'. In work with Kevin Walker, he has defined the notion of a 'disklike n-category'. This object allows us to construct a TQFT in dimension $n + 1$, but also to construct higher order invariants containing more information. In technical terms, the blob complex associates a chain complex to each n-manifold, well defined up to homotopy, and the original TQFT vector space is just the 0-th homology of this chain complex. (This construction only generalizes the `space' part of the TQFT, and has nothing to say about the 'spacetime' part.) This project incorporates ideas from the field of homotopy theory into the study of TQFTs. Using the blob complex, Scott has proved a higher dimensional generalization of Deligne's conjecture on the action of the little discs operad on Hochschild cohomology. A 90 page paper **The blob complex** submitted to *Geometry & Topology* introduces the blob complex and proves this generalization. There is also a companion paper **Higher categories, colimits, and the blob complex** to appear in the *Proceedings of the National Academy of Sciences*.

After completing his Miller Fellowship in 2012, Scott will be moving to Canberra, Australia, to take up a position at the Australian National University.

Obituary

Jerrold Marsden (Miller Professor 1981 - 1982), one of the original founders of reduction theory for mechanical systems with symmetry, passed away on September 21, 2010 at the age of 68.