

Math 3325, 2013

Problem Set 4

Discuss in tutorial on Oct 14

1. Let H be an infinite dimensional Hilbert space. Write down an inner product on H that gives a norm inequivalent with the original norm. Is H complete under the norm determined by the new inner product?

2. Show that the norm on the quotient X/Z of a Banach space X by a closed subspace Z ,

$$\|Z + x\| = \inf_{z \in Z} \|x + z\|_X,$$

is complete.

3. Let l be a linear functional on $C([0, 1])$ defined by

$$l(f) = \int_0^1 K(y)f(y) dy$$

where $K(y)$ is some measurable function. Prove that the norm of l is

$$\int_0^1 |K(y)| dy,$$

hence l is a bounded linear functional if and only if $K \in L^1$.

4. Explain why condition (a) in question 2 of assignment 1 is redundant.