Math 3325, 2014

Problem Set 3

Discuss in tutorial on August 25 and September 1

1. (i) Let $f : \mathbb{R} \to \mathbb{R}$ be an even, integrable function. Show that $\hat{f}$ is an even, real-valued function.

(ii) Let $f : \mathbb{R} \to \mathbb{R}$ be an odd, integrable function. Show that $\hat{f}$ is an odd, purely imaginary function.

2. Let T be the operator $(2\pi)^{-n/2}\mathcal{F}$.

(i) Show that, for $k = 0, 1, 2, 3$, there exists a polynomial $p_k(x)$ of degree k and a complex number c_k such that

$$T\big(p_k(x)e^{-x^2/2}\big) = c_k p_k(\xi)e^{-\xi^2/2}.$$

What is the value of c_k?

(ii) Let b be a complex number of norm 1, but not a fourth root of unity. Show that there is no Schwartz function g such that $Tg = bg$. Hint: what is the operator T^4?

3. Let $\phi \in C_c^\infty(\mathbb{R}^n)$ be nonnegative with integral 1. Let

$$\phi_\delta(x) = \delta^{-n}\phi(\frac{x}{\delta}).$$

Show that for every $f \in L^1(\mathbb{R})$, $f * \phi_\delta$ converges to f in L^1 as $\delta \to 0$.

4. Pick a real number randomly (according to the uniform measure) in the interval $[0, 2]$. Do this one million times and let S be the sum of all the numbers. What, approximately, is the probability that $S \geq 1,001,000$? Express as a definite integral of the function $e^{-x^2/2}$.