Week 4 Linear Algebra worksheet
MATH1014
Lay §4.1, §4.2

(1) Consider the following two systems of linear equations:

$$\begin{aligned} 5x + y - 3z &= 0 \\ -9x + 2y + 5z &= 1 \\ 4x + y - 6z &= 9 \end{aligned} \qquad\qquad \begin{aligned} 5x + y - 3z &= 0 \\ -9x + 2y + 5z &= 5 \\ 4x + y - 6z &= 45 \end{aligned}$$

It can be shown that the first system has a solution. **Use this fact** to show the second system must have a solution.

(2) Let H be the set of vectors in $\mathbb{R}^3$ which are orthogonal to some fixed vector $\mathbf{a}$.
  (a) Show that H is a subspace of $\mathbb{R}^3$.
  (b) Let $T : H \to \mathbb{R}$ be the function defined by $T(\mathbf{x}) = \mathbf{x} \cdot [1, 1, 1]^T$. Show that T is a linear transformation.

(3) A function $T : \mathbb{R}^3 \to M_{2\times 2}$ is defined as follows:

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} 0 & a - 2b \\ a - 2b & b - c \end{bmatrix}.$$

  (a) T is a linear transformation. What would you have to show to verify this fact?
  (b) Which, if any, of the following vectors are in $\ker(T)$?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \\ -4 \end{bmatrix}$$

  (c) Find a basis for $\ker(T)$.

(4) Are the polynomials listed below linearly independent in $\mathbb{P}_2$?

$$1 - 3t, \quad 1 + t^2, \quad 1 - 3t + t^2.$$