Week 8 Linear Algebra worksheet
MATH1014

(1) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

and identify the dimension of each eigenspace.

(2) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are eigenvectors corresponding to different eigenvalues of a matrix M, then $\mathbf{v_1} + \mathbf{v_2}$ cannot be an eigenvector.
   (a) Suppose that M is a 2×2 matrix. For several different examples, draw the parallelogram whose sides are $\mathbf{v_1}$ and $\mathbf{v_2}$. Interpret the statement above in terms of this parallelogram.
   (b) Prove the statement for the case when M is a 2×2 matrix.