Week 13 Linear Algebra worksheet MATH1014

Let $\mathbf{u_1} = \begin{bmatrix} 0\\0\\1\\1 \end{bmatrix}$ and $\mathbf{u_2} = \begin{bmatrix} 1\\1\\0\\0 \end{bmatrix}$. Define $W = \text{Span} \{\mathbf{u_1}, \mathbf{u_2}\}$ and

let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation which is orthogonal projection onto W.

- (1) Find vectors $\mathbf{u_3}$ and $\mathbf{u_4}$ so that $\mathcal{U} = {\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}}$ is an orthogonal basis for \mathbb{R}^4 . (You will use this basis in subsequent parts of this problem.)
- (2) Express the vector $\begin{bmatrix} 1\\1\\1\\1 \end{bmatrix}$ in \mathcal{U} coordinates. Why will everyone

in the tutorial get the same answer?

- (3) What is $T(\mathbf{u_1})$?
- (4) What is $T(\mathbf{u_3})$?
- (5) Find the matrix M_T which represents T with respect to \mathcal{U} coordinates.