Distance workshop

(1) Find the distance from a point P_0 to a line L.

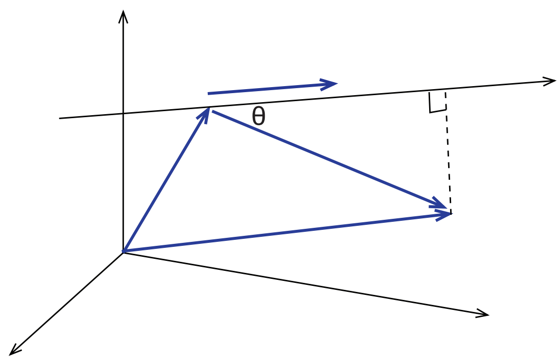
(a) Describe L using a direction vector $\mathbf{v}$ and a specific point P_1 on L.

(b) Let P_2 be the point on L closest to P_0.

(c) Let $\mathbf{r_0}$ be the vector from the origin to P_0.

(d) Let $\mathbf{r_1}$ be the vector from the origin to P_1.

(e) Label the picture.



(f) Use trigonometric identities to describe relationships between as many lengths as you can.

(g) Solve for the distance from P_0 to L.

(2) Find the distance from the point $P_0 = (1, 1, -1)$ to the line L of intersection between the planes

$$x + y + z = 1, \quad 2x - y - 5z = 1.$$

(a) Explain why the direction vector of L is $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2$, where $\mathbf{n}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{n}_2 = 2\mathbf{i} - \mathbf{j} - 5\mathbf{k}$.

(b) Find $\mathbf{v}$.

(c) Pick $P_1 = (1, \frac{-1}{4}, \frac{1}{4})$ on the line. How far is P_1 from the closes point to P_0 on L?

(d) What is the distance from P_0 to each of the two planes?

(3) If L_1 and L_2 are parallel lines which don't intersect, find the distance between them.

Hint: pick an arbitrary point on each line and connect them by a vector. How does the distance relate to this vector? Draw a picture!