## Week 4 Linear Algebra worksheet MATH1014 Lay §4.1, §4.2

(1) Consider the following two systems of linear equations:

$$5x + y - 3z = 0$$
  $5x + y - 3z = 0$   
 $-9x + 2y + 5z = 1$   $-9x + 2y + 5z = 5$   
 $4x + y - 6z = 9$   $4x + y - 6z = 45$ 

It can be shown that the first system has a solution. **Use this fact** to show the second system must have a solution.

- (2) Let H be the set of vectors in  $\mathbb{R}^3$  which are orthogonal to some fixed vector  $\mathbf{a}$ .
  - (a) Show that H is a subspace of  $\mathbb{R}^3$ .
  - (b) Let  $T: H \to \mathbb{R}$  be the function defined by  $T(\mathbf{x}) = \mathbf{x} \cdot [1, 1, 1]^T$ . Show that T is a linear transformation.
- (3) A function  $T: \mathbb{R}^3 \to M_{2\times 2}$  is defined as follows:

$$T\left(\left[\begin{array}{c} a\\b\\c \end{array}\right]\right) = \left[\begin{array}{cc} 0 & a-2b\\a-2b & b-c \end{array}\right].$$

- (a) T is a linear transformation. What would you have to show to verify this fact?
- (b) Which, if any, of the following vectors are in ker(T)?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 4 \\ 2 \\ -4 \end{bmatrix}$$

- (c) Find a basis for ker(T).
- (4) Are the polynomials listed below linearly independent in  $\mathbb{P}_2$ ?

$$1 - 3t$$
,  $1 + t^2$ ,  $1 - 3t + t^2$ .