

Week 8 Linear Algebra worksheet  
MATH1014

- (1) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

and identify the dimension of each eigenspace.

- (2) If  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are eigenvectors corresponding to different eigenvalues of a matrix  $M$ , then  $\mathbf{v}_1 + \mathbf{v}_2$  cannot be an eigenvector.
- (a) Suppose that  $M$  is a  $2 \times 2$  matrix. For several different examples, draw the parallelogram whose sides are  $\mathbf{v}_1$  and  $\mathbf{v}_2$ . Interpret the statement above in terms of this parallelogram.
- (b) Prove the statement for the case when  $M$  is a  $2 \times 2$  matrix.