Week 10 Linear Algebra worksheet
MATH1014

Suppose a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ acts as follows:

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{\mathcal{E}}\right) = \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{\mathcal{E}}\right), \quad T\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}_{\mathcal{E}}\right) = \left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}_{\mathcal{E}}\right),$$

$$T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}_{\mathcal{E}}\right) = \left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}_{\mathcal{E}}\right).$$

(1) Suppose that A is the standard matrix for T. Why is A is diagonalisable?
(2) Choose a basis for $\mathbb{R}^3$ and write down the matrix for T with respect to this basis. (Hint: there is a "best" choice!)
(3) Compare your answer to the standard matrix for the following linear transformation $P : \mathbb{R}^3 \to \mathbb{R}^3$:
- P sends $\mathbf{e_1}$ to $\mathbf{0}$,
- P sends $\mathbf{e_2}$ to $\mathbf{e_2}$,
- P sends $\mathbf{e_3}$ to $\mathbf{e_3}$.

Try to use this comparison to give a geometric description of T.