

Week 12 Linear Algebra worksheet solutions
MATH1014

- (1) Recall the definition of an *orthogonal matrix*. If U is orthogonal, what's special about $U^T U$? $U U^T$?
- (2) Now we'll try to generalise this. Find an example of orthonormal vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in \mathbb{R}^5 .
- (3) If Q is the matrix whose columns are the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ from the previous part, what is $Q^T Q$? $Q Q^T$?
- (4) Repeat Part 2, finding a new set of orthonormal vectors. This will define a different matrix for Part 3, which we'll call P . Check that $Q^T Q = P^T P$ and that $Q Q^T = P P^T$.
- (5) How would you find the orthogonal complement of $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?