

Week 13 Linear Algebra worksheet  
MATH1014

Let  $\mathbf{u}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ . Define  $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$  and

let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  be the linear transformation which is orthogonal projection onto  $W$ .

- (1) Find vectors  $\mathbf{u}_3$  and  $\mathbf{u}_4$  so that  $\mathcal{U} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is an orthogonal basis for  $\mathbb{R}^4$ . (You will use this basis in subsequent parts of this problem.)

- (2) Express the vector  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$  in  $\mathcal{U}$  coordinates. Why will everyone in the tutorial get the same answer?

- (3) What is  $T(\mathbf{u}_1)$ ?  
(4) What is  $T(\mathbf{u}_3)$ ?  
(5) Find the matrix  $M_T$  which represents  $T$  with respect to  $\mathcal{U}$  coordinates.