

Your essay for Math 3325 should be about 5-8 pages long, if typed, and consist mostly of words! Of course, you can have mathematical symbols and equations, but try to keep it readable, and be sure to explain the idea behind any proof you give.

Make sure that the essay has a clear goal (e.g. to explain the ideas behind a significant theorem, and give a sketch of the proof), and that the goal is explicitly stated at the beginning of the essay.

The audience can be assumed to be other members of the Math 3325 class. That is, assume that the reader has background equivalent to a typical Math 3325 student.

### **Some suggested essay topics:**

- Dirichlet problem - boundary behaviour. Many sources, for example Gilbarg-Trudinger...
- Dimension of fractals. See Hutchinson, Fractals and Self-similarity, Indiana University Mathematics Journal, vol 30 (1981), p713–
- Noncomplemented subspaces. Rudin, Functional analysis, p125.
- Wavelets. Existence and construction of wavelet bases. Ingrid Daubechies, Ten lectures on wavelets.
- Inversion of the Fourier transform on  $L^p$ . Look at Terence Tao, Recent Progress on the restriction conjecture, Lecture 5.
- Trace class operators and Lidskii's theorem. Start with Reed and Simon, Methods of modern mathematical physics vol. 1.
- The Hilbert transform. See Stein, Singular Integrals and Differentiability properties of functions.
- The restriction theorem for the Fourier transform. See Tomas, A restriction theorem for the Fourier Transform, Bulletin of the American Mathematical Society 81 (1975), p477.
- Construction of Brownian motion. See M. E. Taylor, Partial Differential Equations vol 2, chapter 11,
- Ergodic theory. See the text, chapter 6, and Reed and Simon, Methods of modern mathematical physics vol. 1, section VII.4.
- Dvoretzky's theorem. Try Y. Gordon, Gaussian processes and almost spherical sections of convex bodies, Ann. Probab. 16 (1988), no. 1, 180–188.
- Fixed point theorems, for example the Leray-Schauder fixed point theorem, or the Kakutani fixed point theorem.
- Bochner's theorem on the Fourier transform of a positive measure.
- The Fast Fourier transform (FFT).