

Your essay for Math 3325 should be about 5-8 pages long, if typed, and consist mostly of words! Of course, you can have mathematical symbols and equations, but try to keep it readable, and be sure to explain the idea behind any proof you give.

Make sure that the essay has a clear goal (e.g. to explain the ideas behind a significant theorem, and give a sketch of the proof), and that the goal is explicitly stated at the beginning of the essay.

The audience can be assumed to be other members of the Math 3325 class. That is, assume that the reader has background equivalent to a typical Math 3325 student.

### Some suggested essay topics:

- Dirichlet problem - boundary behaviour. Many sources, for example Gilbarg-Trudinger...
- Noncomplemented subspaces. Rudin, Functional analysis, p125.
- Wavelets. Existence and construction of wavelet bases. Ingrid Daubechies, Ten lectures on wavelets.
- Inversion of the Fourier transform on  $L^p$ . Look at Terence Tao, Recent Progress on the restriction conjecture, Lecture 5.
- Trace class operators and Lidskii's theorem. Start with Reed and Simon, Methods of modern mathematical physics vol. 1.
- The Hilbert transform. See Stein, Singular Integrals and Differentiability properties of functions.
- Bochner's theorem, and the Fourier-Stieltjes transform.
- The restriction theorem for the Fourier transform. See Tomas, A restriction theorem for the Fourier Transform, Bulletin of the American Mathematical Society 81 (1975), p477.
- Construction of Brownian motion. See M. E. Taylor, Partial Differential Equations vol 2, chapter 11,
- Ergodic theory. See the text, chapter 6, and Reed and Simon, Methods of modern mathematical physics vol. 1, section VII.4.
- Dvoretzky's theorem. Try Y. Gordon, Gaussian processes and almost spherical sections of convex bodies, Ann. Probab. 16 (1988), no. 1, 180–188.
- Fixed point theorems, for example the Leray-Schauder fixed point theorem, or the Kakutani fixed point theorem.
- Bochner's theorem on the Fourier transform of a positive measure.
- The Fast Fourier transform (FFT).

- $C^*$ -algebras, the Gelfand transform; commutative  $C^*$ -algebras are continuous functions on a compact Hausdorff space; the Gelfand-Naimark theorem.
- von Neumann algebras; the double commutant theorem.
- The Banach-Tarskii paradox, amenability. (See Terry Tao's "An epsilon of room", for example.)
- Haar measure on Lie groups, the Peter-Weyl theorem.