

Math 3325, 2017

Problem Set 3

Discuss in tutorial on August 21 and 28

1. (i) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be an even, integrable function. Show that  $\hat{f}$  is an even, real-valued function.

(ii) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be an odd, integrable function. Show that  $\hat{f}$  is an odd, purely imaginary function.

2. Let  $T$  be the operator  $(2\pi)^{-n/2}\mathcal{F}$ .

(i) Show that, for  $k = 0, 1, 2, 3$ , there exists a polynomial  $p_k(x)$  of degree  $k$  and a complex number  $c_k$  such that

$$T(p_k(x)e^{-x^2/2}) = c_k p_k(\xi)e^{-\xi^2/2}.$$

What is the value of  $c_k$ ?

(ii) Let  $b$  be a complex number of norm 1, but not a fourth root of unity. Show that there is no Schwartz function  $g$  such that  $Tg = bg$ . Hint: what is the operator  $T^4$ ?

3. Let  $\phi \in C_c^\infty(\mathbb{R}^n)$  be nonnegative with integral 1. Let

$$\phi_\delta(x) = \delta^{-n}\phi\left(\frac{x}{\delta}\right).$$

Show that for every  $f \in L^1(\mathbb{R})$ ,  $f * \phi_\delta$  converges to  $f$  in  $L^1$  as  $\delta \rightarrow 0$ .

4. Pick a real number randomly (according to the uniform measure) in the interval  $[0, 2]$ . Do this one million times and let  $S$  be the sum of all the numbers. What, approximately, is the probability that  $S \geq 1,001,000$ ? Express as a definite integral of the function  $e^{-x^2/2}$ .